

Aperture Photometry on PLATO Target Stars

5th forum of Action Fédératrice Etoiles (AFE)



Laboratoire d'Études Spatiales et d'Instrumentation en Astrophysique



ESCOLA
POLITÉCNICA
DA USP



UNIVERSITY OF SÃO PAULO



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MEUDON - 13/11/2017

Overview

- The PLATO Mission
 - Main science goals
 - Instrument characteristics
 - Science requirements
- Data processing chain: instrument calibration and correction algorithms
- PLATO's on-board photometry problematic
- Contaminant star flux on the on-board photometry
- Mitigation of contaminant flux in the on-board lightcurves

The PLATO Mission: overview

- A medium size (M3) mission of the ESA's Cosmic Vision Program expected to be launched by 2026 in L2 orbit

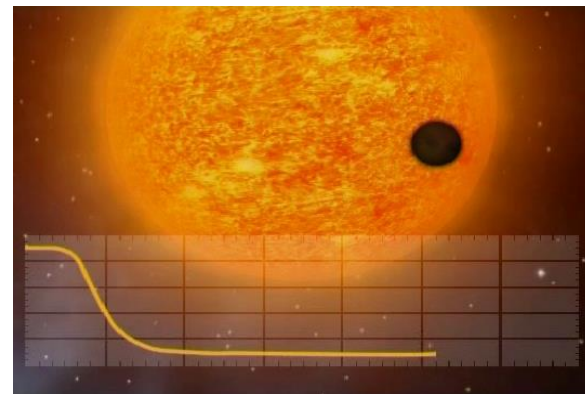
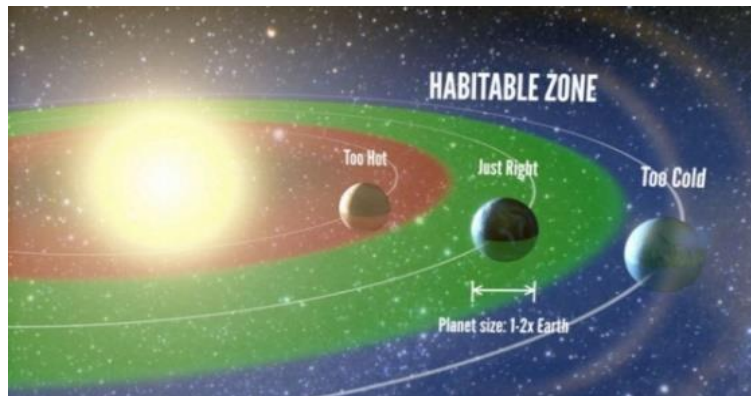
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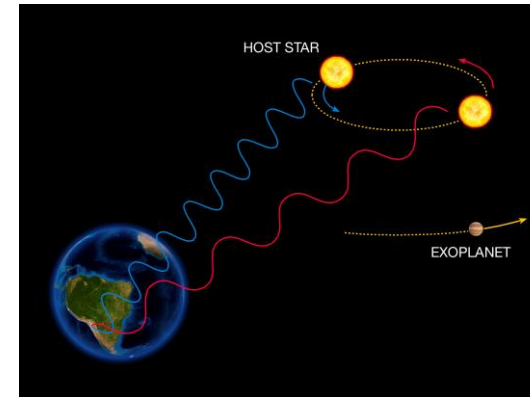
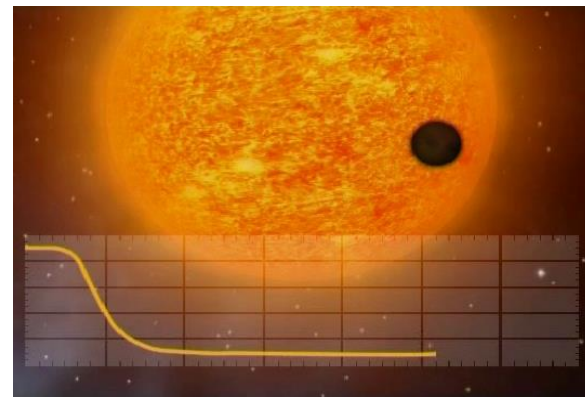
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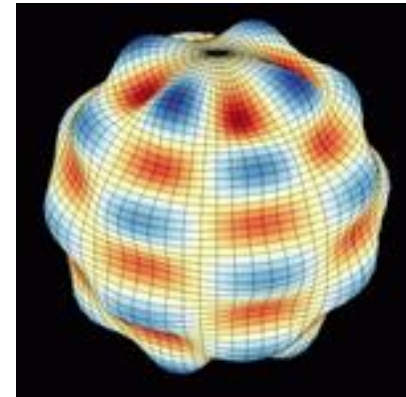
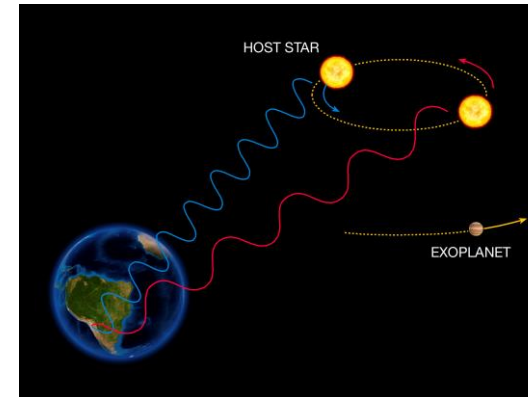
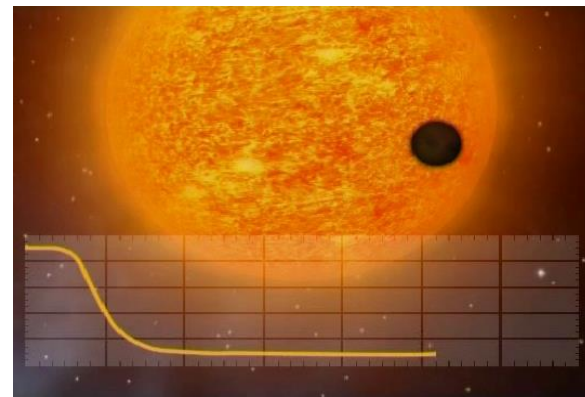
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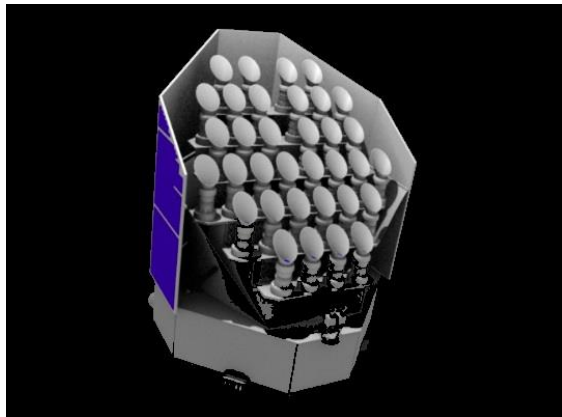
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 - Stellar masses and radii determination to an accuracy of a few percent from asteroseismology
 - Stellar age determination to an accuracy better than 10% from asteroseismology



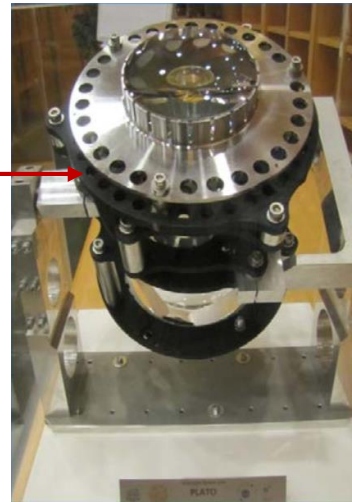
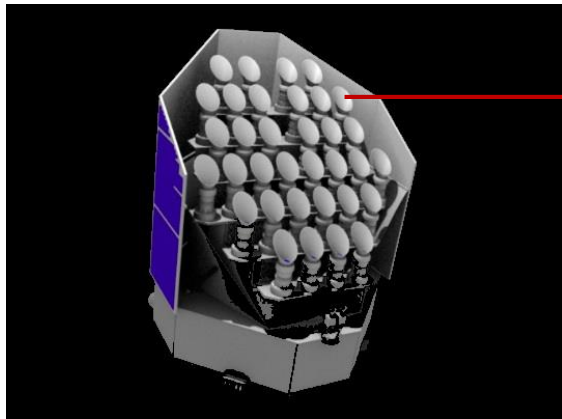
The PLATO Mission: instrument

- Instrument: multi-telescope concept
 - 2 “fast” (2.5 seconds cadence) telescopes for Attitude and Orbit Control System
 - 24 “normal” (25 seconds cadence) telescopes for the core science
 - Field of View (FoV): $\cong 2300 \text{ deg}^2$



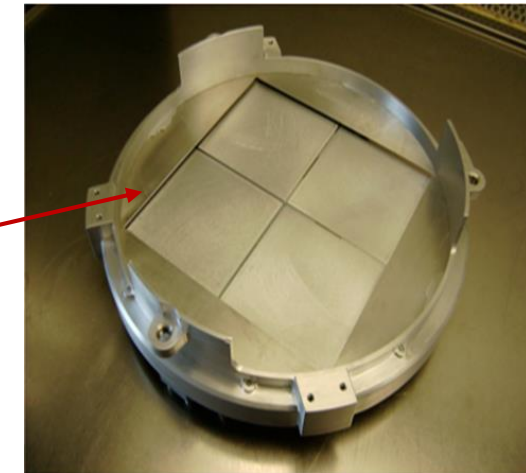
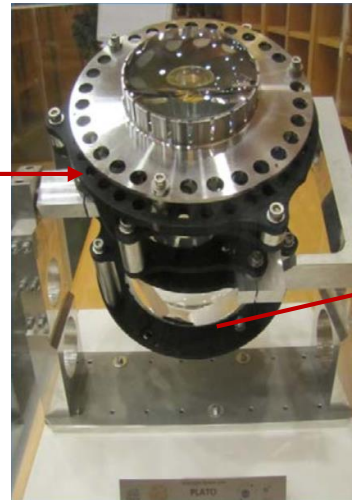
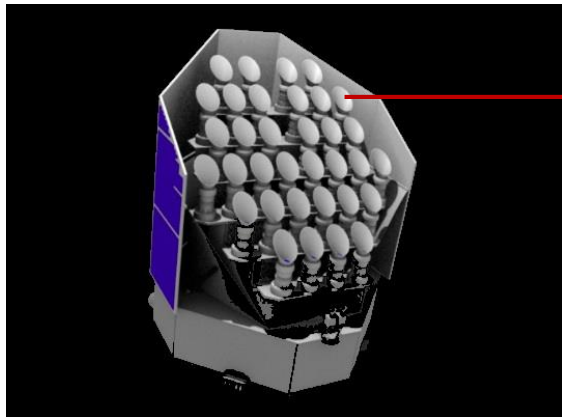
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 - Characteristics of each telescope (camera):
 - $\cong 1100 \text{ deg}^2$ FoV with 120mm pupil and fully dioptric design (6 lenses)
 - Four CCDs (detectors) with $4510 \times 4510 \text{ } 18\mu\text{m}$ pixels each
 - Wavelength band: 500–1000 nm



The PLATO Mission: science requirements

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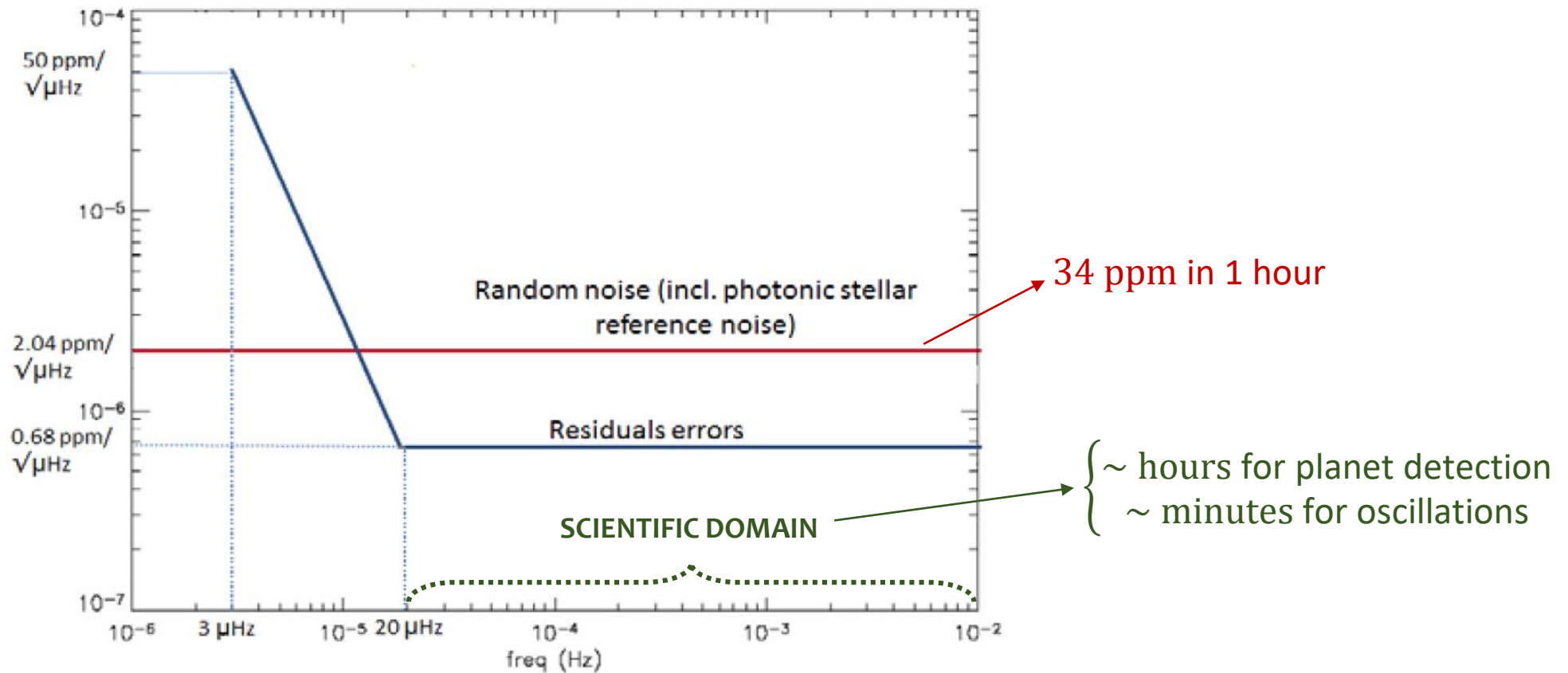
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- Asteroseismology noise requirements
 - 34 ppm in 1 hour to detect short life-time solar-like oscillations of dwarf stars at $m_v \leq 11$
- Planetary transit noise requirements
 - 80 ppm in 1 hour to detect Earth-like planets orbiting the habitable zone of $m_v \leq 13$ stars

The PLATO Mission: science requirements

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- Systematic error residuals must be limited to 1/3 of random noise



Data processing chain: instrument calibration and correction algorithms

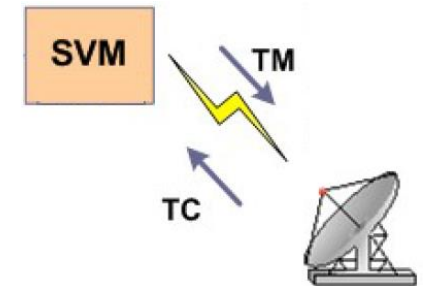
- Main sources of noise and systematic errors
 - Detectors: Charge Transfer Inefficiency (CTI), brighter-fatter effect
 - Pointing: jitter
 - Platform: thermoelastic distortion (multi-telescopes approach)
 - Natural: kinematic aberration, sky background, contaminant stars

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- In order to derive lightcurves within noise requirements, several on-board and on-ground calibration/correction algorithms are needed

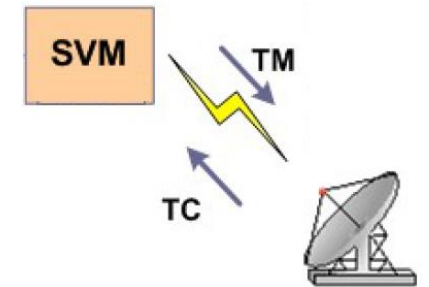
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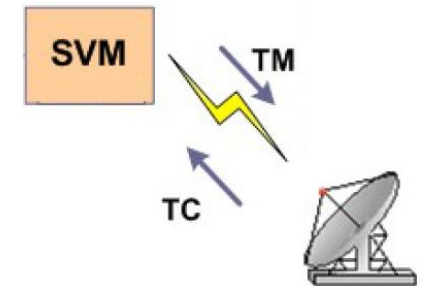
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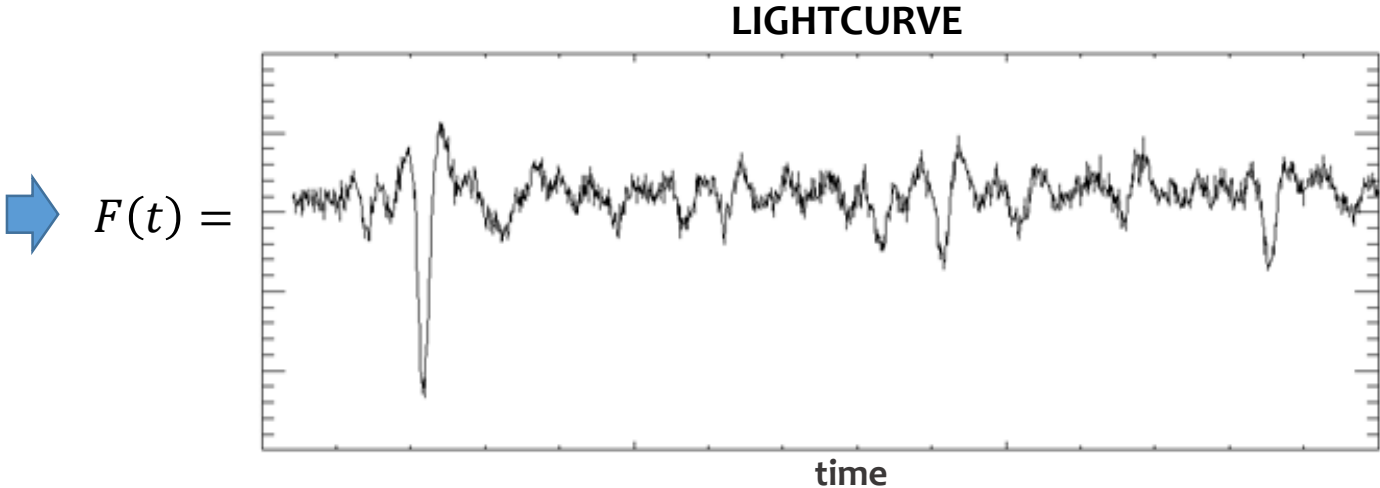
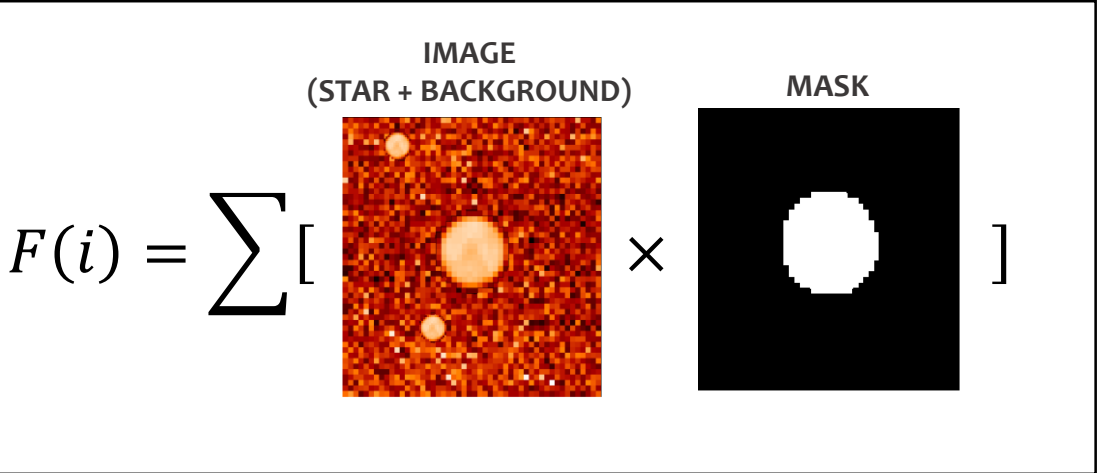
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- In order to derive lightcurves within noise requirements, several on-board and on-ground calibration/correction algorithms are needed
- Constraints: limitation of telemetry and on-board CPU budgets
- On-board photometry required for the majority of the target stars
 - Aperture (mask-based) photometry presents the best compromise between Noise-To-Signal Ratio (NSR) and computational cost



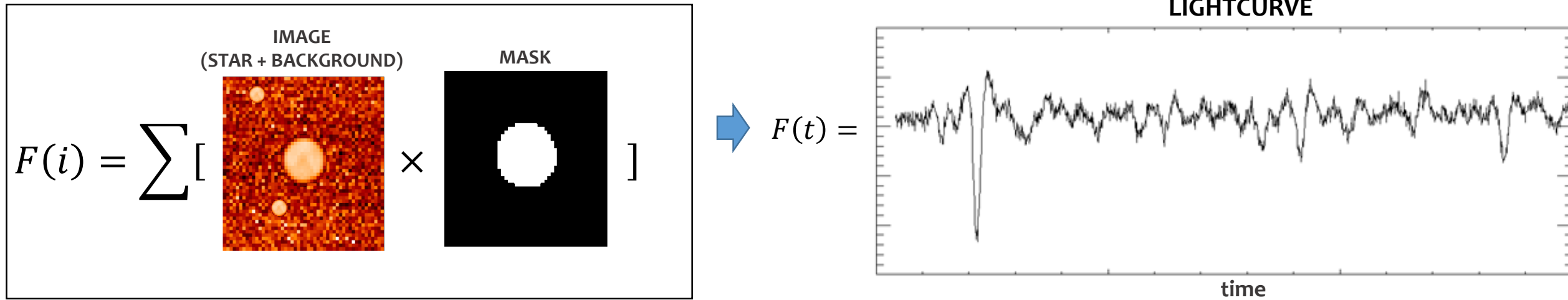
Satellite on-board photometry: aperture masks

- Principle



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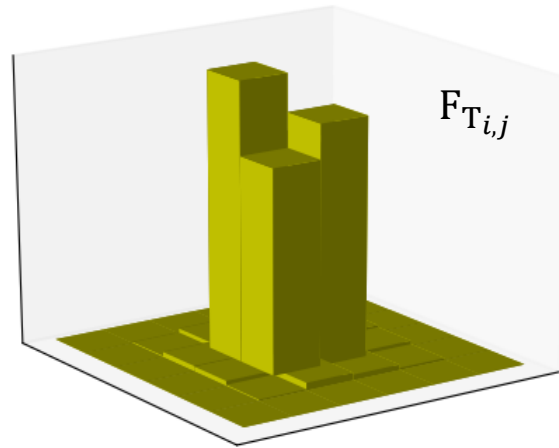


- Objective: develop optimized aperture photometry masks models
 - Provide the lowest possible NSR
 - Minimize the presence of contaminant flux
 - Constrain payload CPU, memory and telemetry
 - Limit as well as possible the residuals to be corrected on-ground

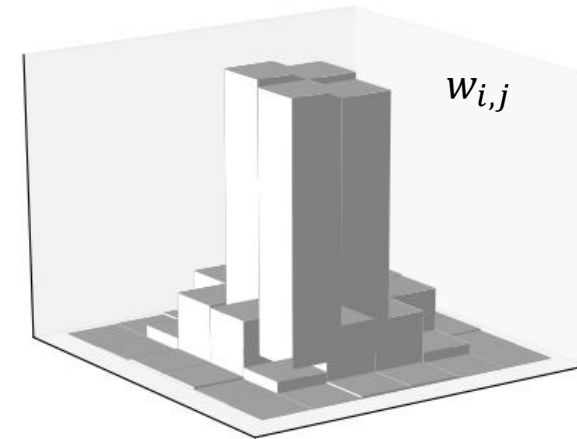
Measuring aperture masks performance

- Signal-to-Noise Ratio (SNR)

TARGET STAR SIGNAL



MASK



$$\text{SNR} = \frac{\sum_{i,j} (F_{T,i,j} \times w_{i,j})}{\sqrt{\sum_{i,j} (F_{T,i,j} \times w_{i,j}^2) + \sum_{i,j} (B_{i,j} \times w_{i,j}^2) + \sum_{i,j} (R_{i,j}^2 \times w_{i,j}^2)}}$$

$F_{T,i,j}$ = target star flux at pixel (i, j)

$w_{i,j}$ = mask weight at pixel (i, j)

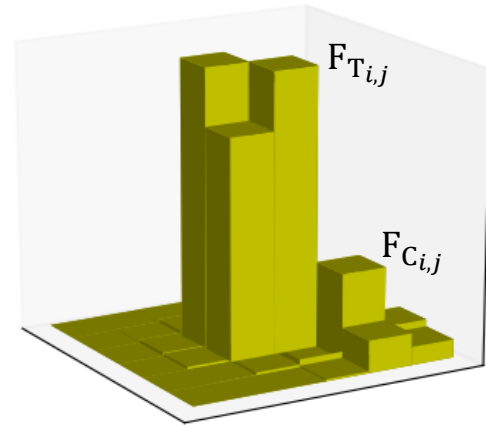
$B_{i,j}$ = background noise at pixel (i, j)

$R_{i,j}$ = readout noise at pixel (i, j)

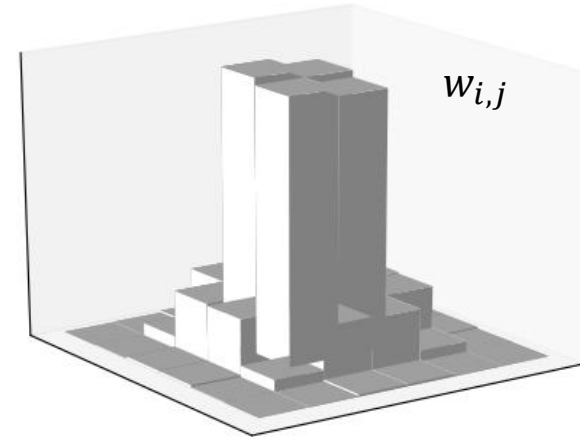
Measuring aperture masks performance

- Contamination rate (τ_C)

TARGET + CONTAMINANT STARS
SIGNAL



MASK



$$\tau_C = \frac{\sum_{i,j} (F_{C_{i,j}} \times w_{i,j})}{\sum_{i,j} [(F_{T_{i,j}} + F_{C_{i,j}}) \times w_{i,j}]}$$

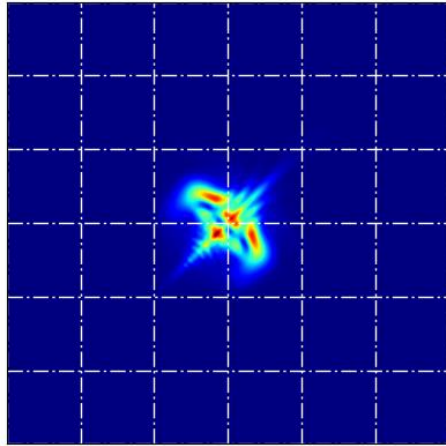
$(0 \leq \tau_C \leq 1)$

$F_{C_{i,j}}$ = contaminant star flux at pixel (i, j)
 $F_{T_{i,j}}$ = target star flux at pixel (i, j)
 $w_{i,j}$ = mask weight at pixel (i, j)

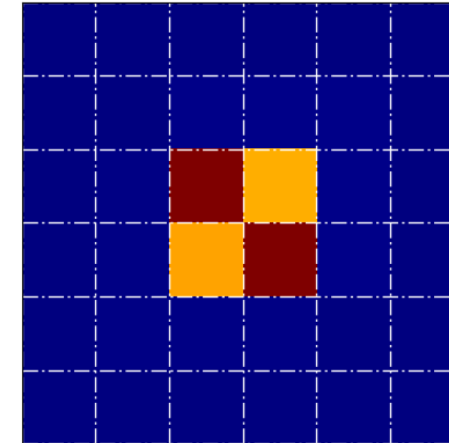
PSF Enclosure energy

- At detector level, star flux are translated into low spatial resolution images called imagettes

PSF reaching detector

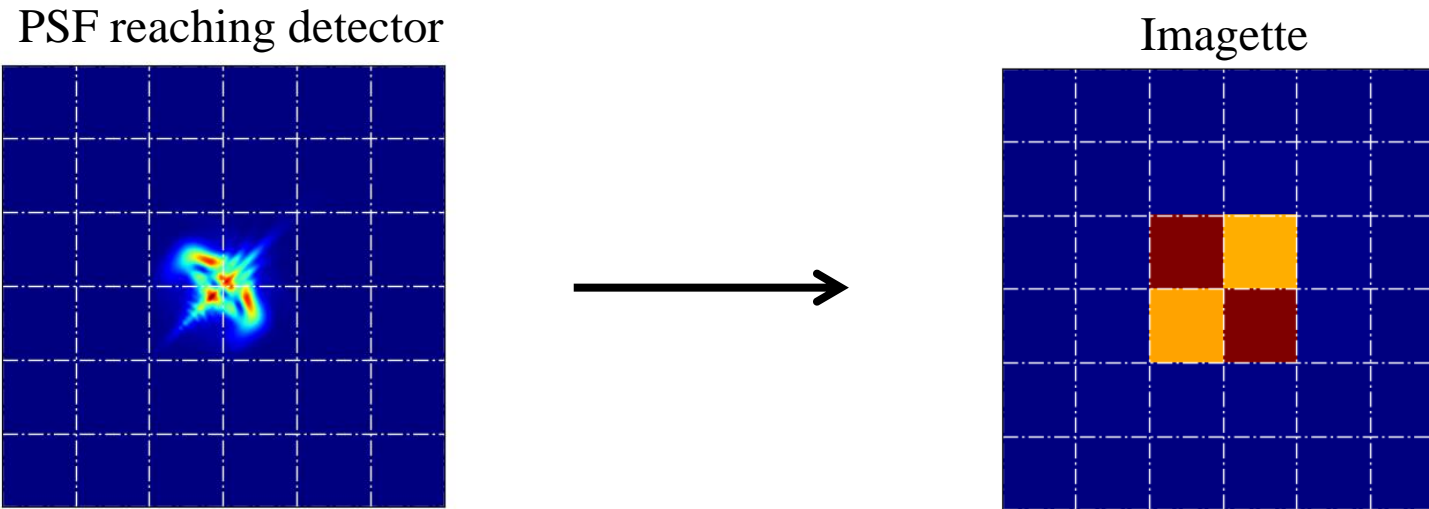


Imagette



PSF Enclosure energy

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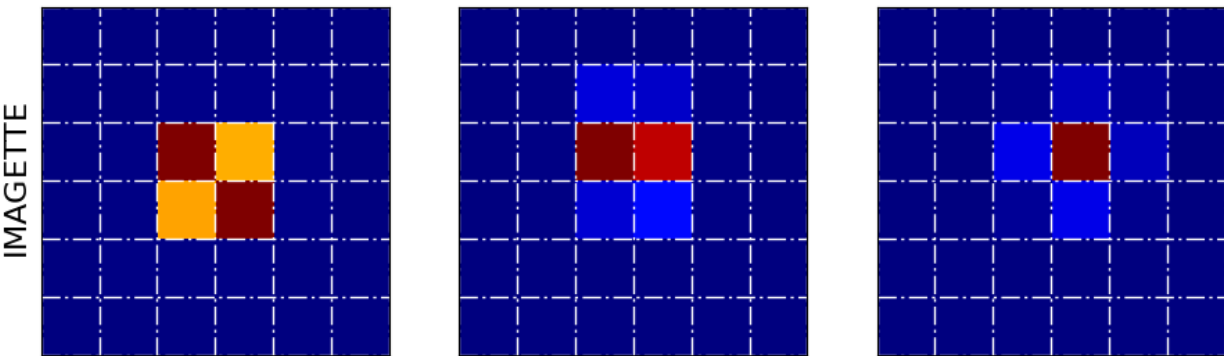
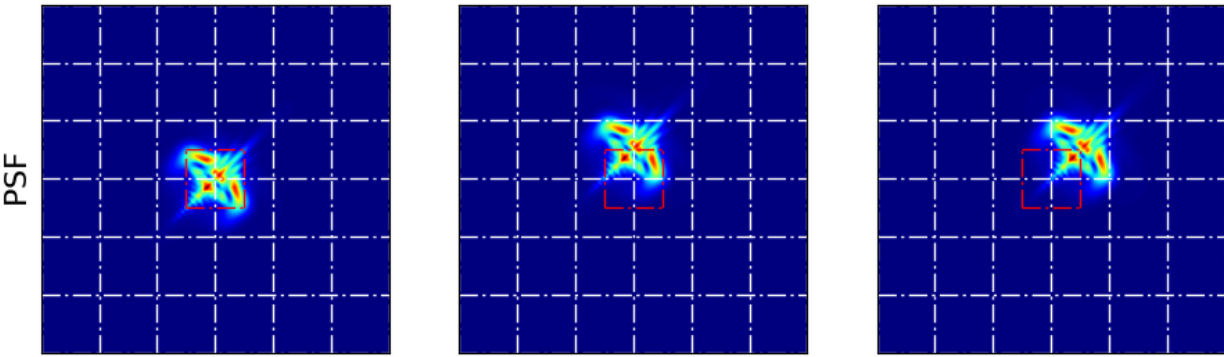


- 90% of the PSF energy is concentrated within about 4 pixels of the CCDs
 - Increases the number of observed stars
 - Reduces overlap of target and contaminant flux
 - Increases SNR (in a large sense)
 - Aperture photometry performance: higher sensibility to star position drift

Long-term drift of star centroids

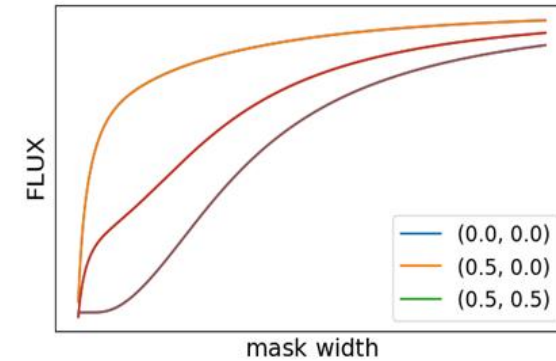
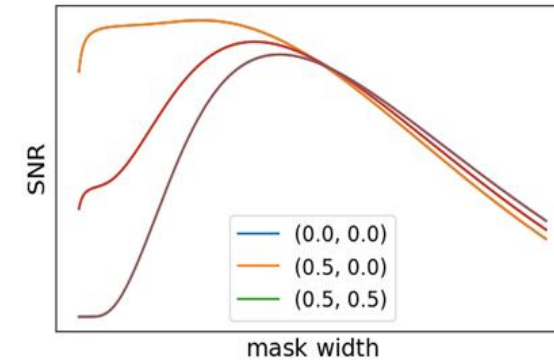
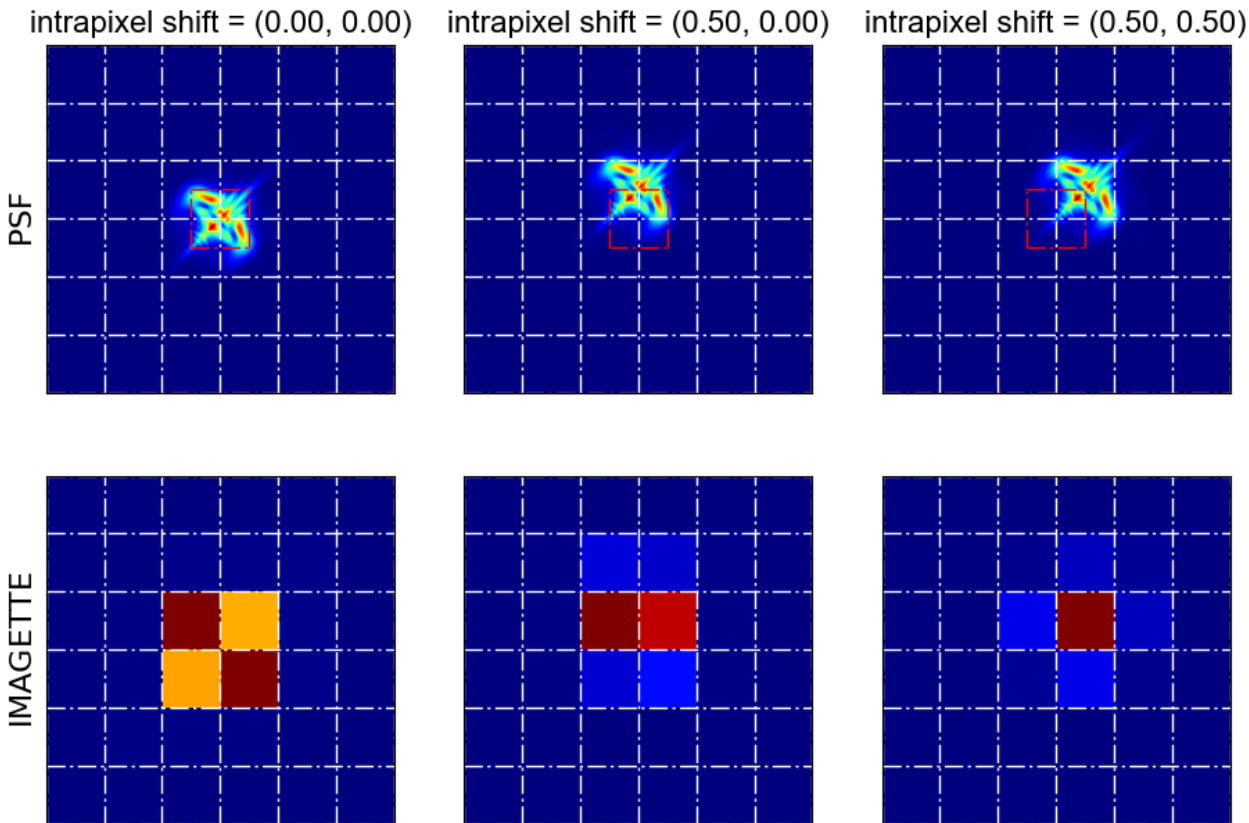
- Thermoelastic distortion and kinematic aberration may primarily account for star centroid displacements that can reach up to 1.3 pixel over 3 months (worst case)

intrapixel shift = (0.00, 0.00) intrapixel shift = (0.50, 0.00) intrapixel shift = (0.50, 0.50)



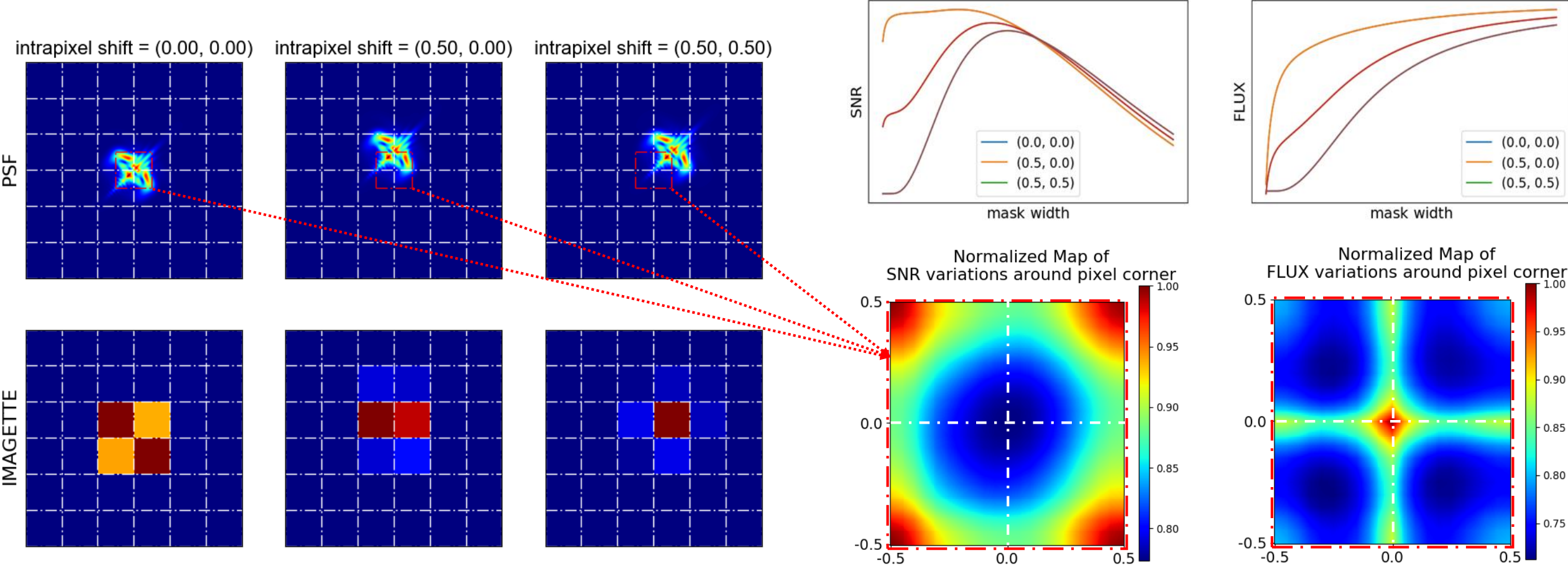
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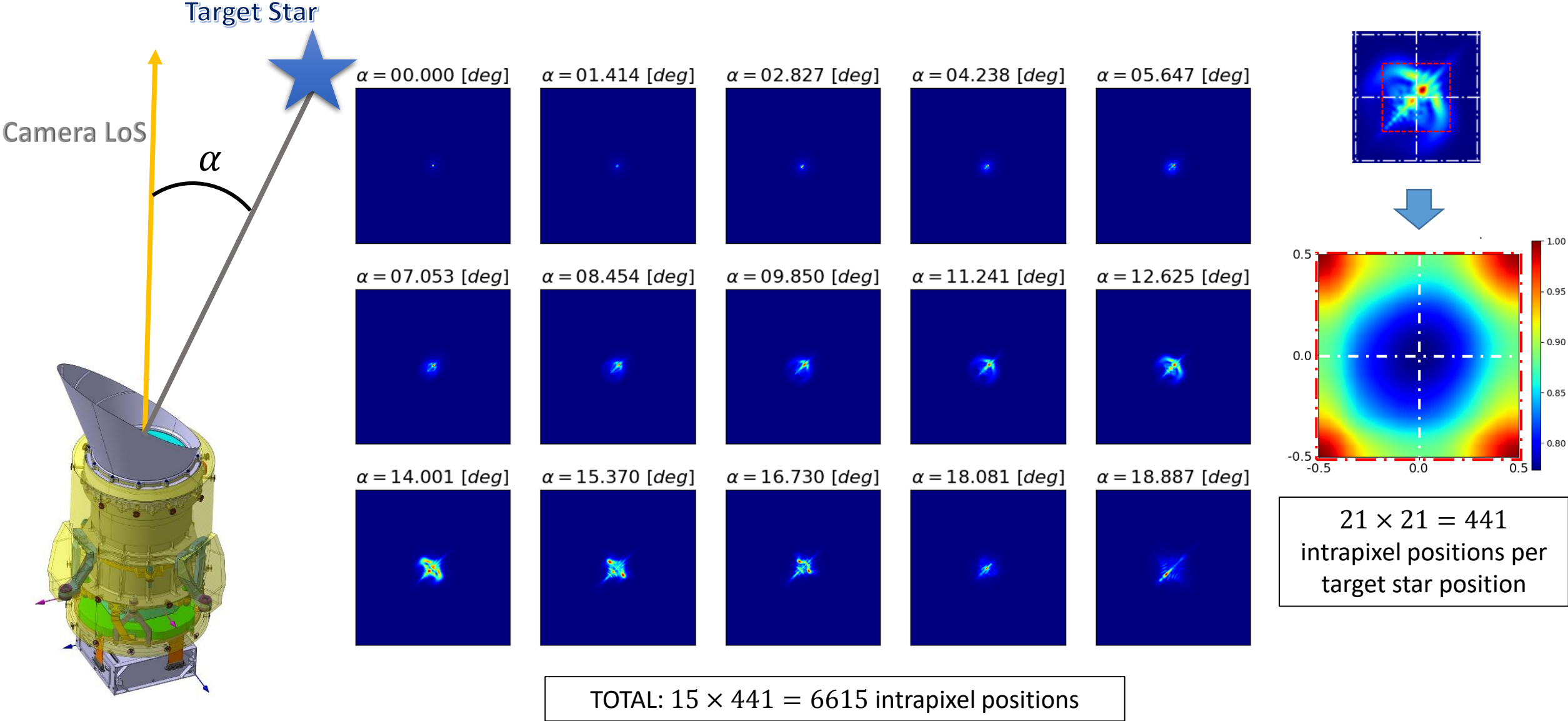


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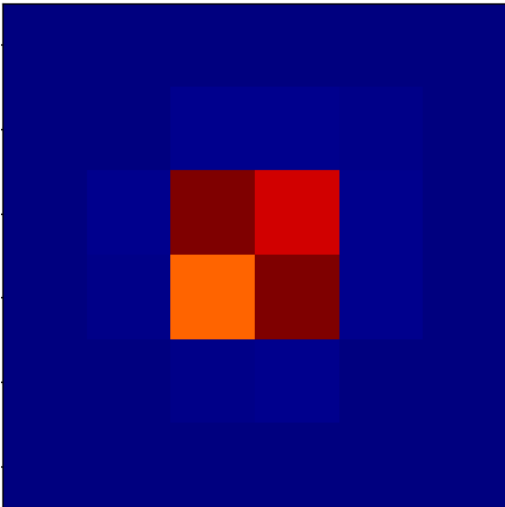
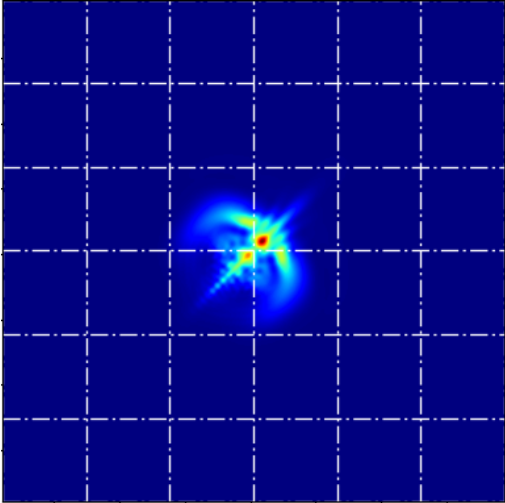


Analysis of masks performances: methodology



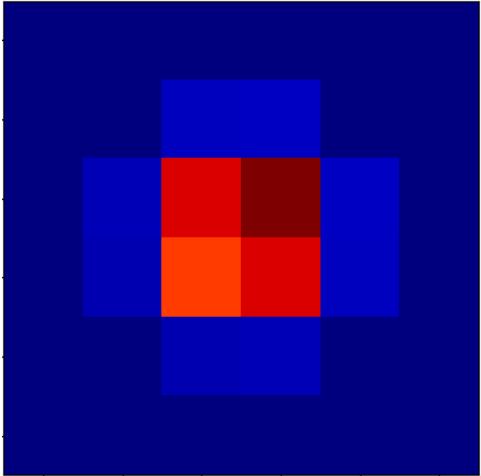
Analysis of masks performances: methodology

TARGET STAR ONLY



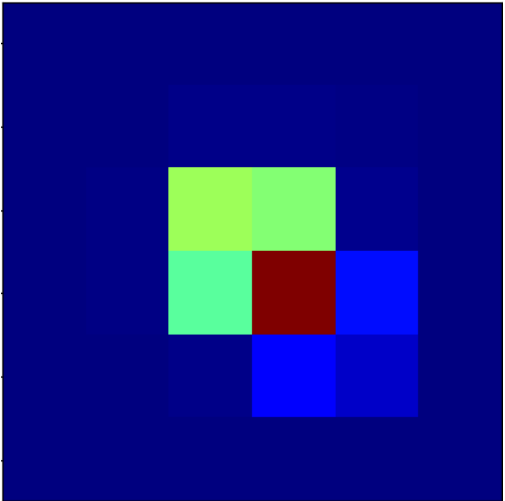
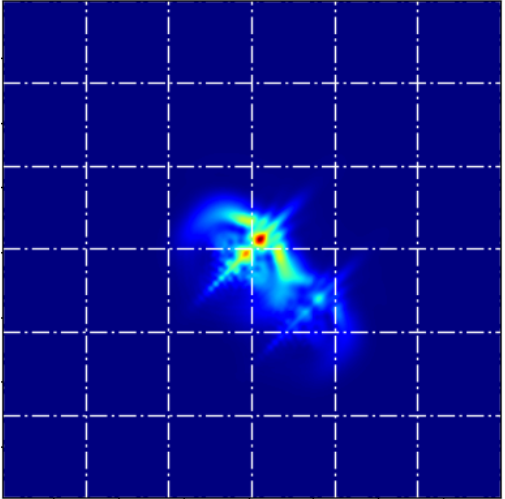
UTILIZED
TO
COMPUTE

MASK



APPLIED TO

TARGET + CONTAMINANT STARS

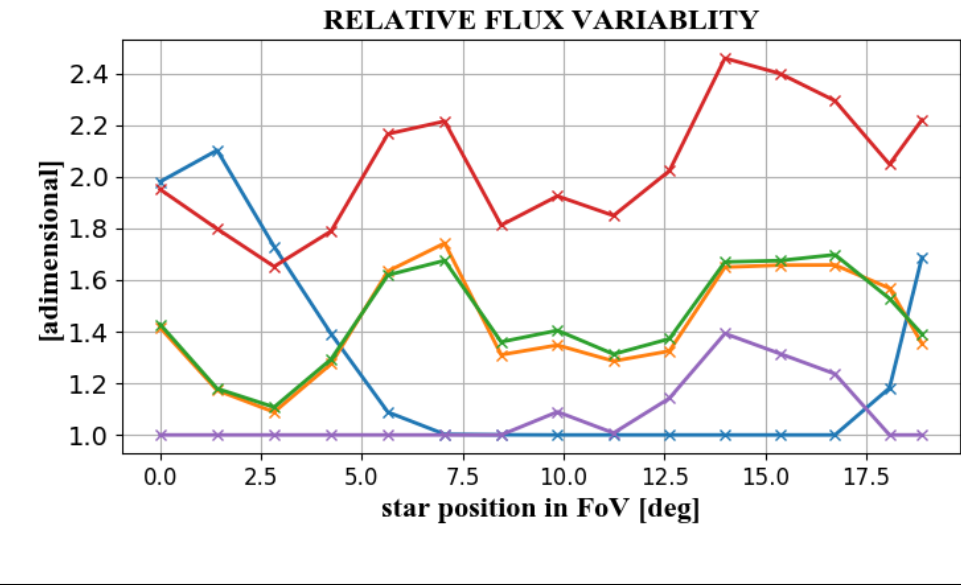
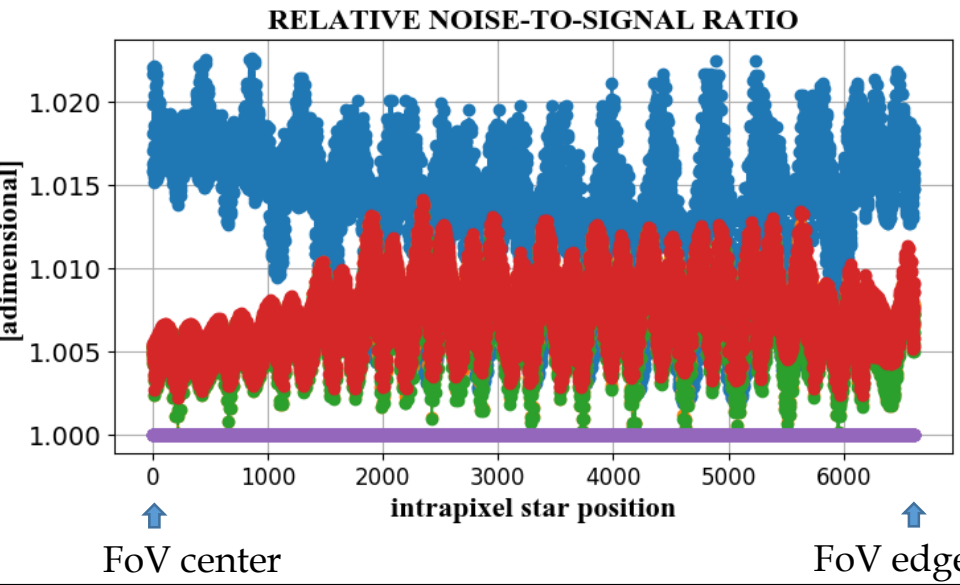
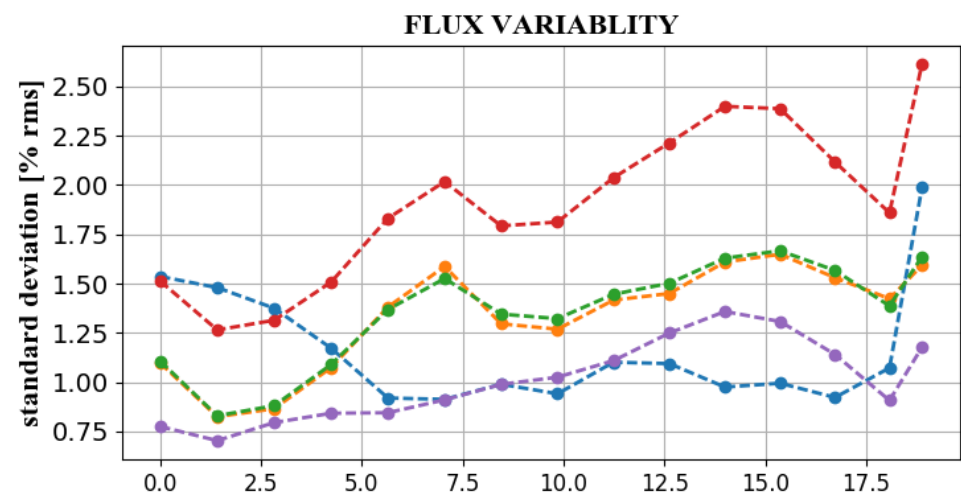
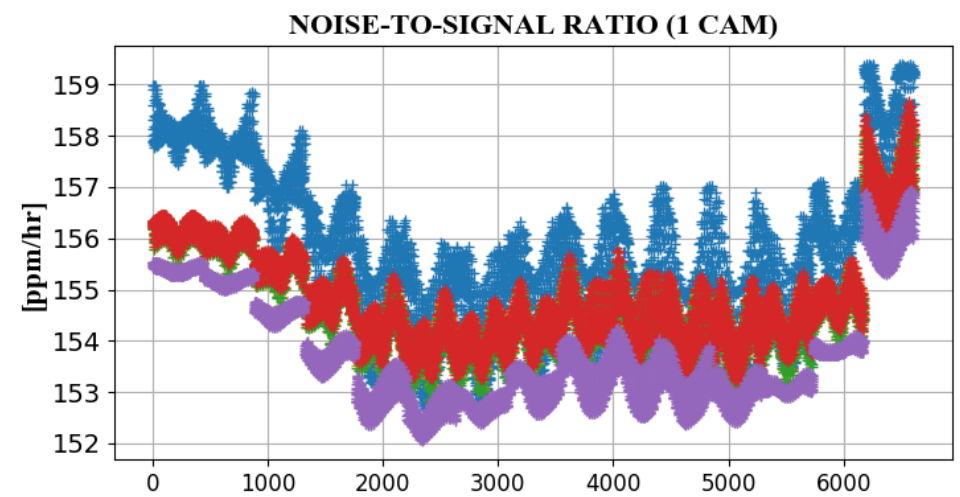


Target star parameters:
 $m_v = 10.7$ and $T = 6000[K]$
Contaminant star parameters:
 $m_v = 11.7$ and $T = 6000[K]$
Evaluated distances:
[0.5, 1.0, 1.5, 2.0, 2.5, 3.0] pixel(s)

Analysis of masks performances: results

Target star : $m_V = 10.7$; $T = 6000[K]$; No Contaminant star

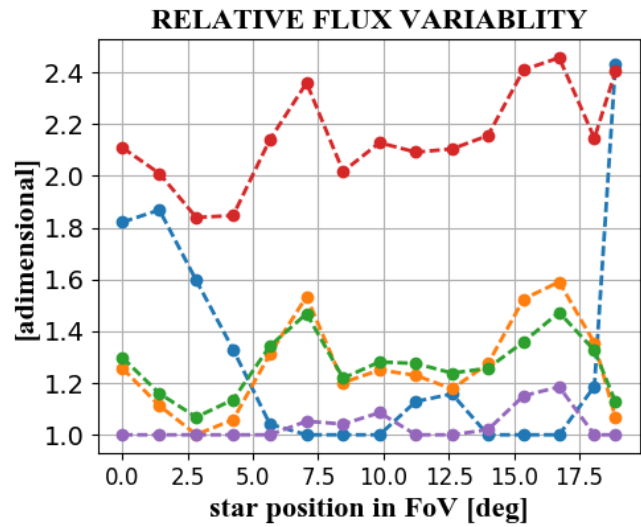
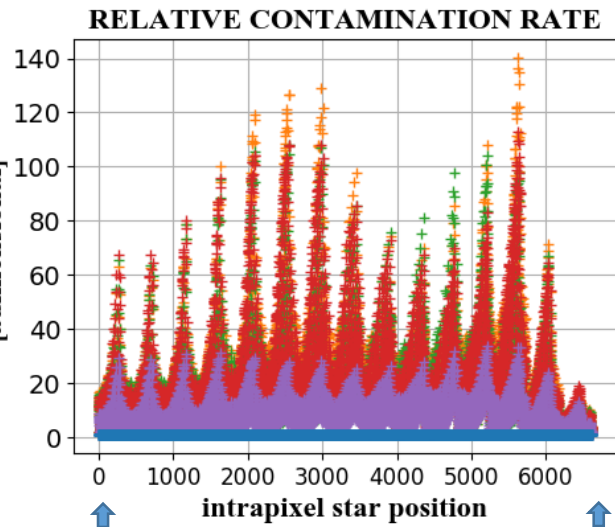
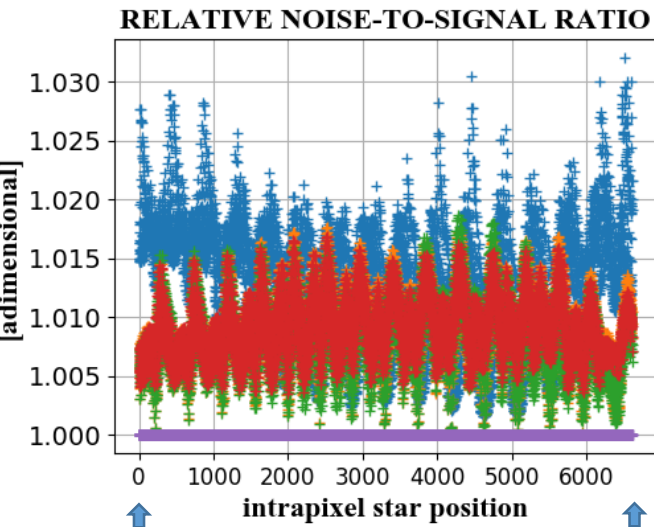
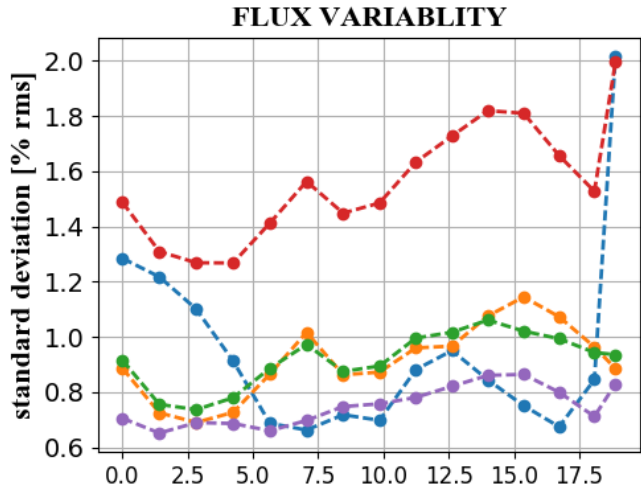
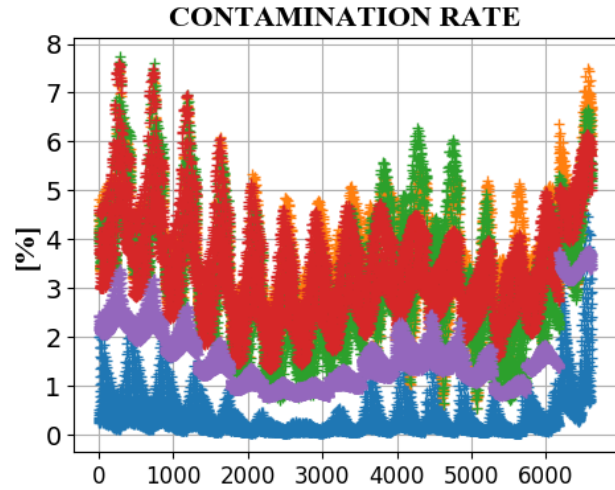
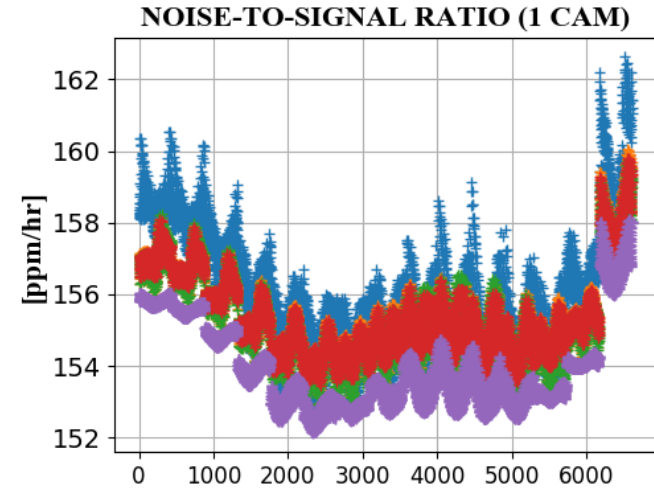
- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask



Analysis of masks performances: results

Target star : $m_v = 10.7$; $T = 6000[K]$; Contaminant star : $m_v = 11.7$; $T = 6000[K]$
 Euclidian distance between target and contaminant stars : 2.5 pixels

- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask



FoV center

FoV edge

FoV center

FoV edge

Analysis of masks performances

- Overall results

- Target star: $m_v = 10.7$, $T = 6000[K]$; Contaminant star: $m_v = 11.7$, $T = 6000[K]$

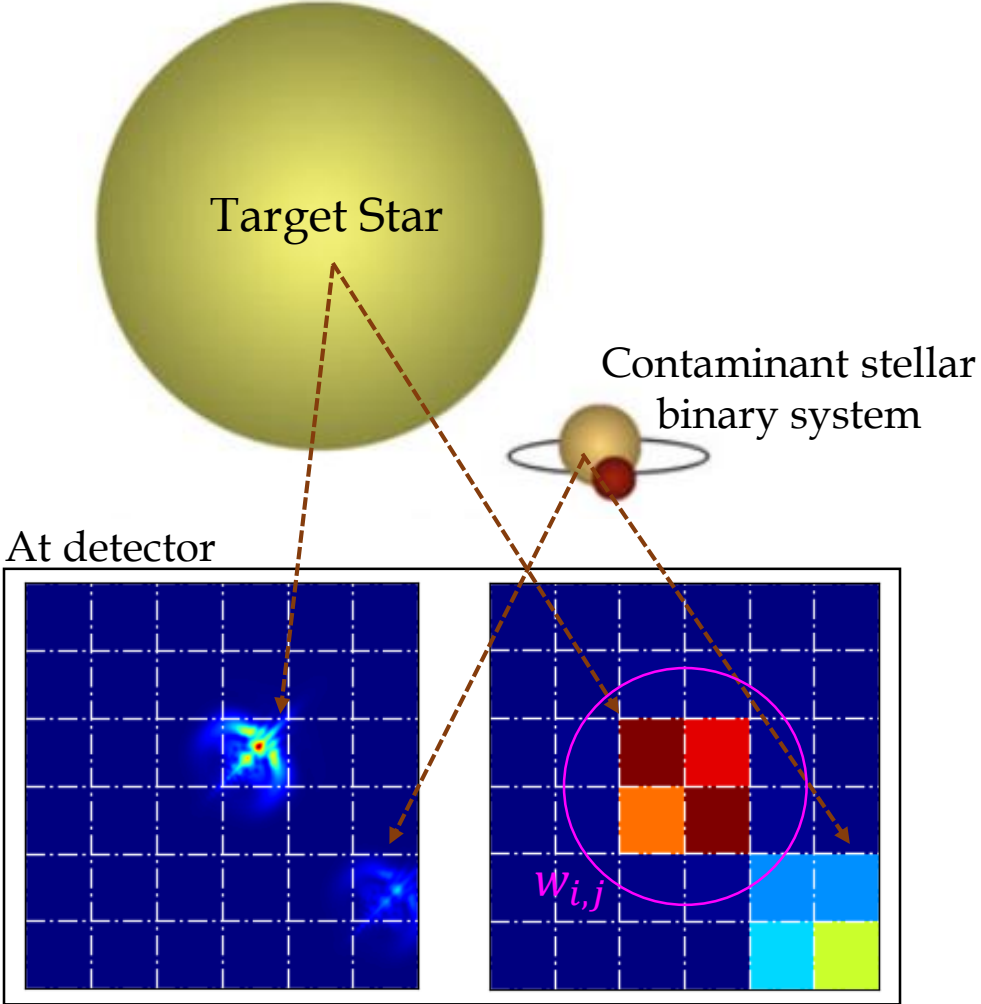
PERFORMANCE PARAMETER	DISTANCE (target to contaminant) [pixels]	MASK MODEL				
		Optimal NSR Binary	Optimal NSR Weights	Optimal NSR Symmetric Gaussian	Optimal NSR Asymmetric Gaussian	Fixed Width Symmetric Gaussian
Relative NSR	∞	1.00 - 1.02	1.00	1.00 - 1.01	1.00 - 1.01	1.00 - 1.01
	3.0	1.00 - 1.02	1.00	1.00 - 1.01	1.00 - 1.01	1.00 - 1.01
	2.5	1.00 - 1.03	1.00	1.00 - 1.02	1.00 - 1.02	1.00 - 1.02
	2.0	1.00 - 1.08	1.00 - 1.01	1.00 - 1.03	1.00 - 1.03	1.00 - 1.03
	1.5	1.00 - 1.12	1.00 - 1.03	1.00 - 1.04	1.00 - 1.04	1.00 - 1.04
	1.0	1.00 - 1.12	1.00 - 1.04	1.00 - 1.05	1.00 - 1.05	1.00 - 1.05
	0.5	1.00 - 1.08	1.00 - 1.03	1.00 - 1.04	1.00 - 1.04	1.00 - 1.04
Relative Flux variability (STD column)	∞	1.00 - 2.10	1.00 - 1.39	1.09 - 1.74	1.11 - 1.70	1.65 - 2.46
	3.0	1.00 - 2.16	1.00 - 1.41	1.06 - 1.67	1.10 - 1.60	1.67 - 2.54
	2.5	1.00 - 2.43	1.00 - 1.19	1.00 - 1.59	1.07 - 1.47	1.84 - 2.46
	2.0	1.69 - 5.45	1.00 - 1.12	1.11 - 4.16	1.00 - 4.13	1.82 - 4.67
	1.5	3.31 - 9.29	1.00 - 1.82	1.00 - 1.58	1.00 - 1.69	1.14 - 3.66
	1.0	2.06 - 4.08	1.00 - 2.05	1.00 - 1.14	1.00 - 1.22	1.39 - 2.15
	0.5	1.20 - 2.13	1.00 - 1.29	1.00 - 2.02	1.03 - 2.01	1.44 - 2.37
Relative Contamination rate	∞	-	-	-	-	-
	3.0	1.00 - 1.06	1.00 - 25.63	2.14 - 90.46	1.57 - 81.36	1.62 - 77.03
	2.5	1.00 - 1.30	1.00 - 33.59	1.34 - 140.2	1.32 - 107.79	1.03 - 113.02
	2.0	1.00 - 1.86	1.00 - 34.14	1.02 - 115.03	1.00 - 99.85	1.00 - 101.33
	1.5	1.00 - 2.00	1.00 - 15.58	1.00 - 30.82	1.00 - 28.62	1.00 - 28.71
	1.0	1.00 - 1.73	1.00 - 3.10	1.00 - 3.54	1.00 - 3.49	1.00 - 3.67
	0.5	1.00 - 1.32	1.00 - 1.24	1.00 - 1.39	1.00 - 1.37	1.00 - 1.38

Conclusions from the presented analysis

- Noise-to-Signal Ratio: typically quite low differences among mask models
- Contamination rate: compared to binary masks, larger masks present poor performance when contaminant star is $\sim 1.5 - 3.0$ pixel distant from target star
- Price to pay for having narrow masks: a few percent higher NSR typically
- Price to pay for having larger masks: up to orders of magnitude higher contaminant flux depending on star position in FoV
 - Is it a concern? Yes...

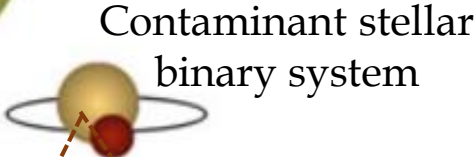
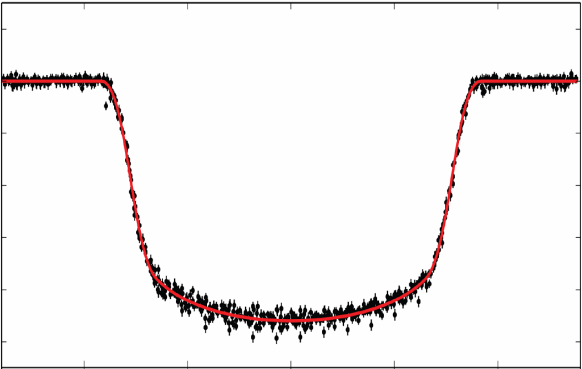
Conclusions from the presented analysis

False-Transit



Conclusions from the presented analysis

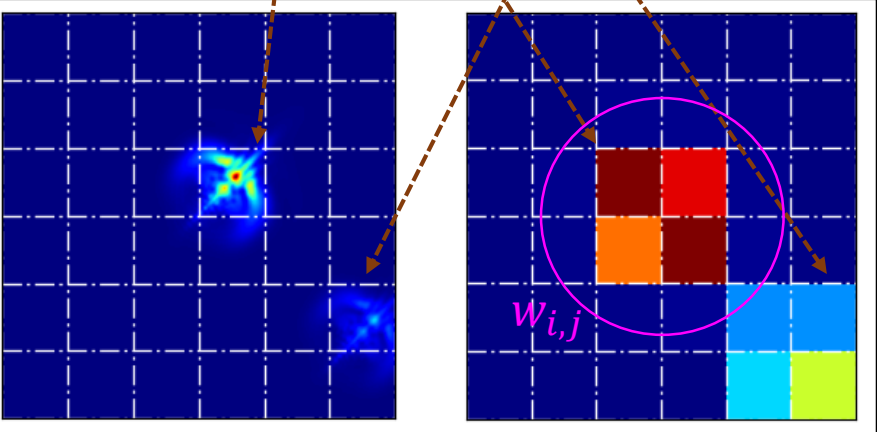
False-Transit



$$\delta_{obs} = \frac{\tau_C}{1 - \tau_C} (1 - 10^{-0.4\delta_{back}})$$

(δ_{back} is given in [mag] units)

At detector

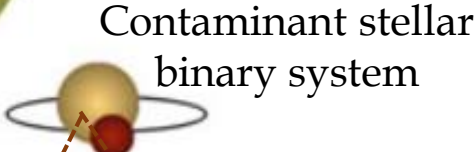
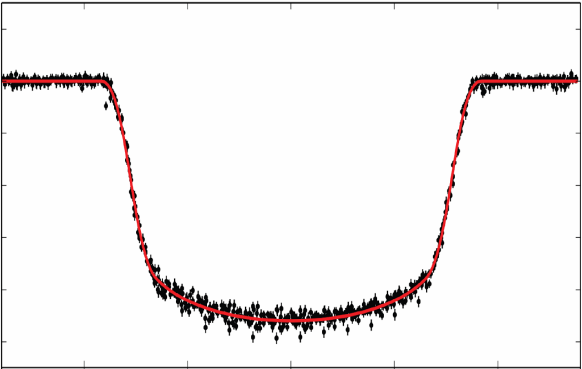


Conclusions from the presented analysis

False-Transit

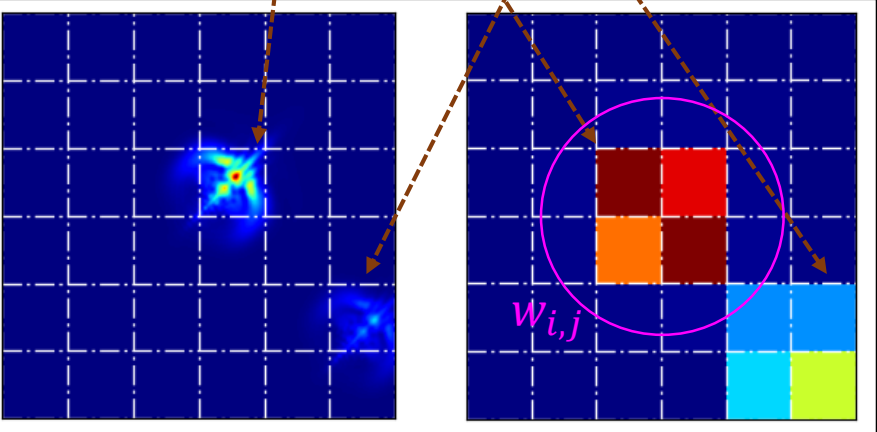


Target Star



Contaminant stellar binary system

At detector



$$\delta_{obs} = \frac{\tau_C}{1 - \tau_C} (1 - 10^{-0.4\delta_{back}})$$

(δ_{back} is given in [mag] units)

δ_{back} [mag]	Typical δ_{obs}		
	Gas giant	Neptunian	Earth
	0.01	0.001	80ppm
	Critical τ_C [%]	Critical τ_C [%]	Critical τ_C [%]
0.2	5.61	0.59	0.048
0.4	3.14	0.32	0.026
0.6	2.30	0.23	0.019
0.8	1.88	0.19	0.015
1.0	1.63	0.17	0.013

Conclusions from the presented analysis

- Contaminant rate is relevant on its own, NSR alone is not enough to guarantee adequate planetary science performance
- On-board photometry optimization shall not be performed only as function of the NSR (only parameter with formal specification)
- There is no silver bullet!
No mask model can simultaneously minimize all performance parameters (NSR, flux variability, contamination rate, jitter etc.)

Strategies for choosing on-board masks to be studied

- 1. Hybrid masks
 - Advantage: reduces contaminant flux and flux discontinuities induced by jitter without sacrificing too much NSR
 - Drawback: sub-optimal performance for all parameters

Strategies for choosing on-board masks to be studied

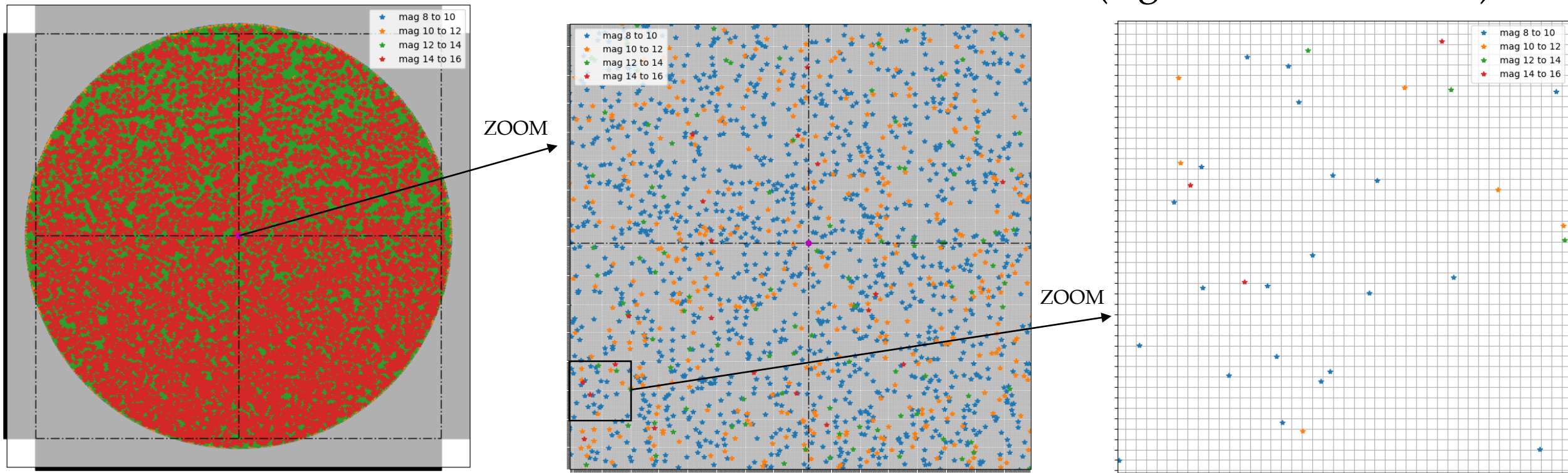
- 1. Hybrid masks
 - Advantage: reduces contaminant flux and flux discontinuities induced by jitter without sacrificing too much NSR
 - Drawback: sub-optimal performance for all parameters
- 2. Multiple masks
 - Advantages: allows to extract the best from each mask model as a function of the contamination scenario
 - Gaia catalogue allows precise identification of contamination conditions, so the “best fit mask” could be determined on-ground prior to the observations
 - Drawback: consumes more on-board resources (to be quantified), need to check feasibility
 - Justifiability

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 - Justifiability
- Obstacle:
 - No formal requirements for flux variability nor contamination rate

Next steps

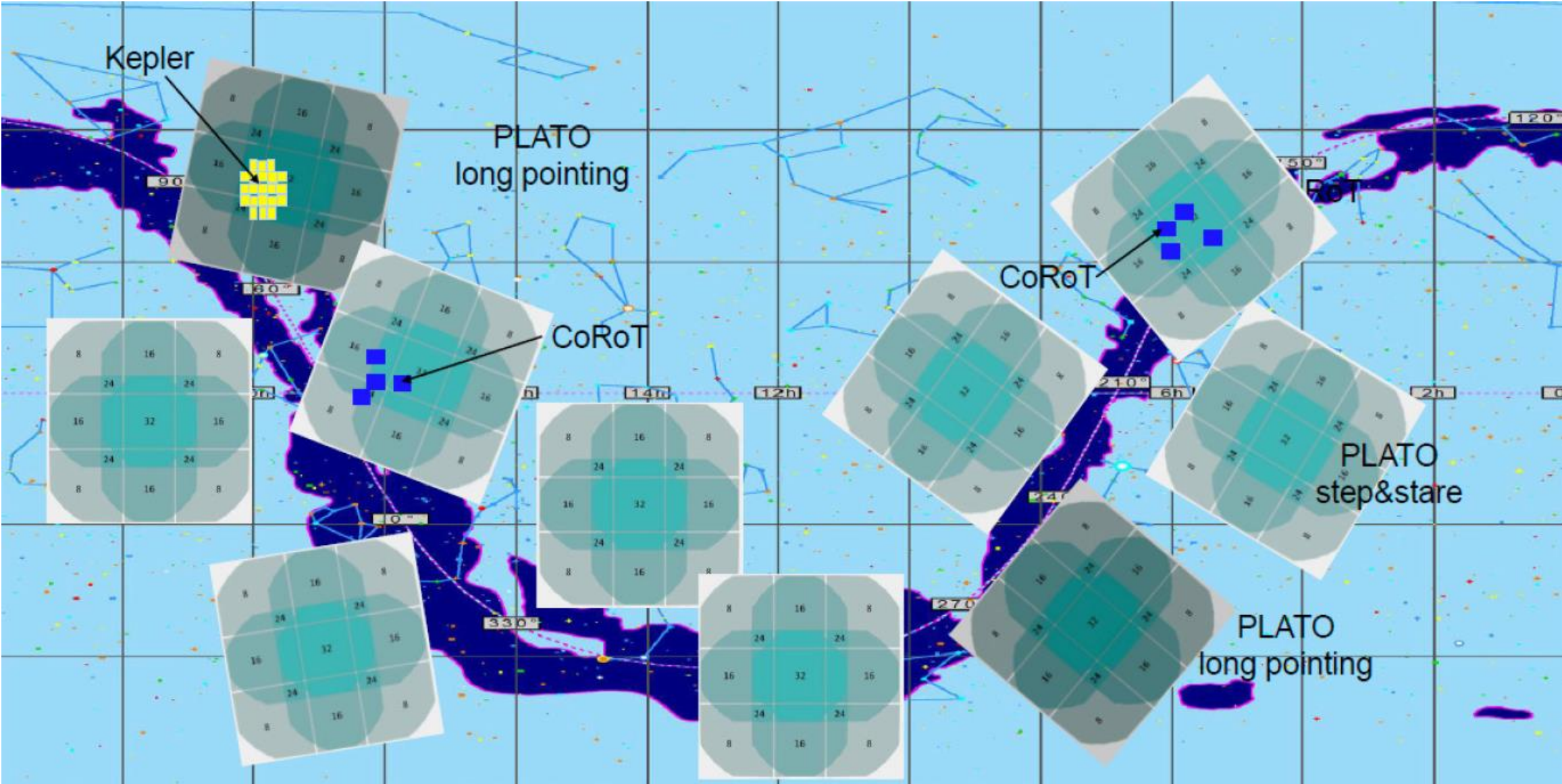
- Study an alternative on-board mask strategy
 - Utilize GAIA catalogue to constrain contamination rate (e.g. first PLATO FoV)



- Quantitatively constrain jitter sensibility and CCD effects in the masks performances analysis

TO BE CONTINUED...

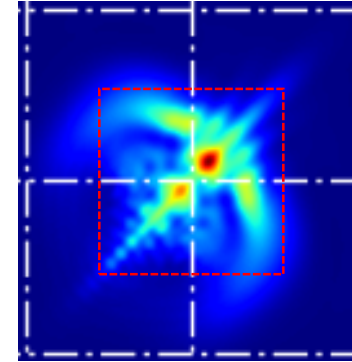
The PLATO Mission: sky coverage



Analysis of masks performances: testing conditions and parameters

- Target star magnitude: 10.7 ; Target star temperature: 6000[K]

- Target star intrapixel positions:
 - Displacement range [-0.5, +0.5] w.r.t. pixel corner
 - Displacement step: 1/20 pixel



- Contaminant star magnitude: 11.7 ; Parasite star temperature: 6000[K]
- Contaminant star distance to target star: [0.5, 1.0, 1,5, 2.0, 2,5, 3.0] pixel(s)

Analysis of masks performances: testing conditions and parameters

- Photometric masks (5 models)
 - Optimal SNR binary
 - Optimal SNR weights
 - Optimal SNR symmetric Gaussian
 - Optimal SNR asymmetric Gaussian
 - Fixed-width symmetric Gaussian

IMAGETTE

MASK

$F_{T,i,j}$

$w_{i,j}$

$F_{T,i,j}$ = target star flux at pixel (i, j)
 $w_{i,j}$ = mask weight at pixel (i, j)
 $B_{i,j}$ = background noise at pixel (i, j)
 $R_{i,j}$ = readout noise at pixel (i, j)

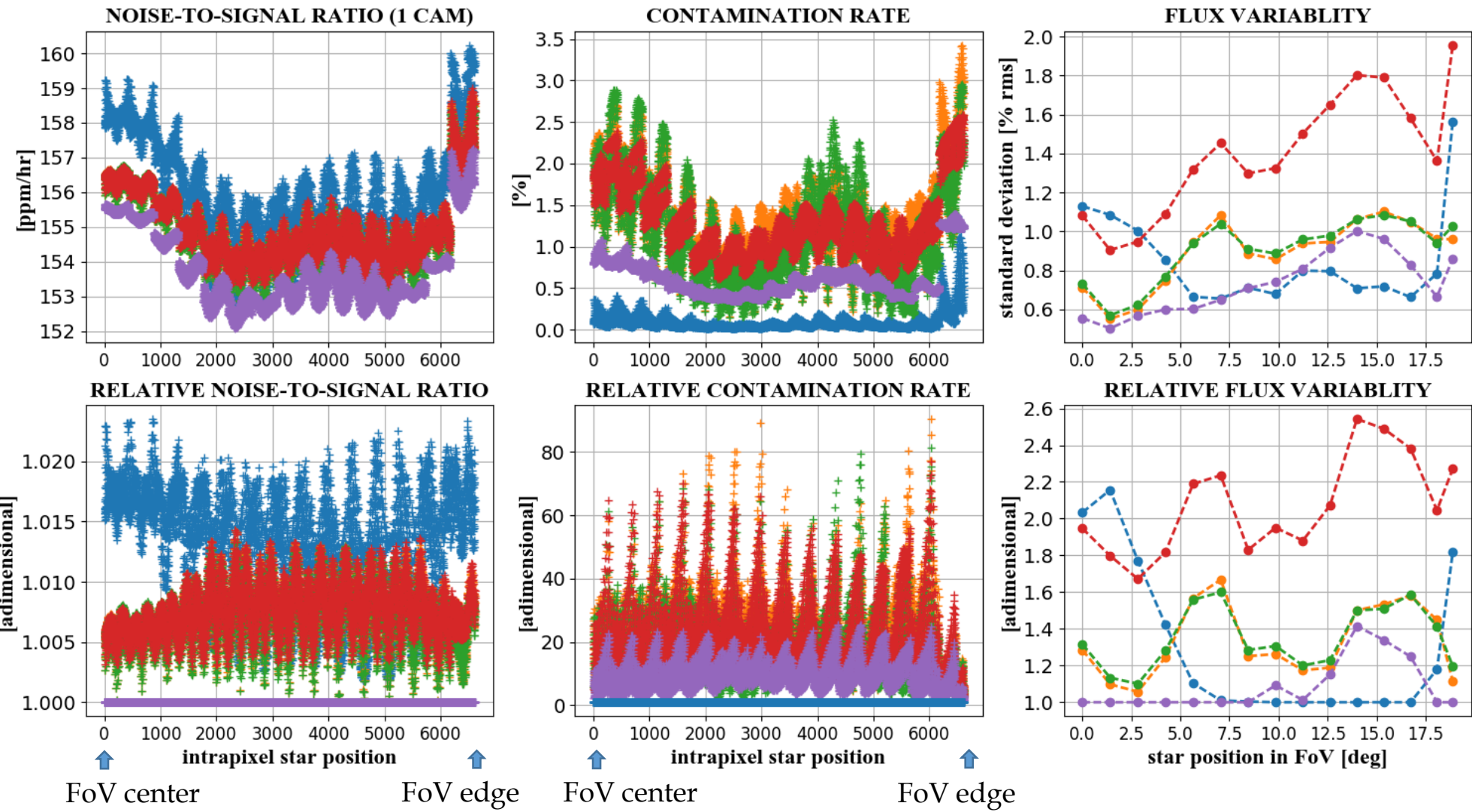
$$SNR = \frac{\sum_{i,j} (F_{T,i,j} \times w_{i,j})}{\sqrt{\sum_{i,j} (F_{T,i,j} \times w_{i,j}^2) + \sum_{i,j} (B_{i,j} \times w_{i,j}^2) + \sum_{i,j} (R_{i,j}^2 \times w_{i,j}^2)}}$$

- Exposure time: 21s ; background noise = 134[e⁻/px/s]; readout noise = 60[e⁻/px]
- No jitter ; no CCD effects (e.g. CTI, Brighter-Fatter, Diffusion)

Analysis of masks performances: results

- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask

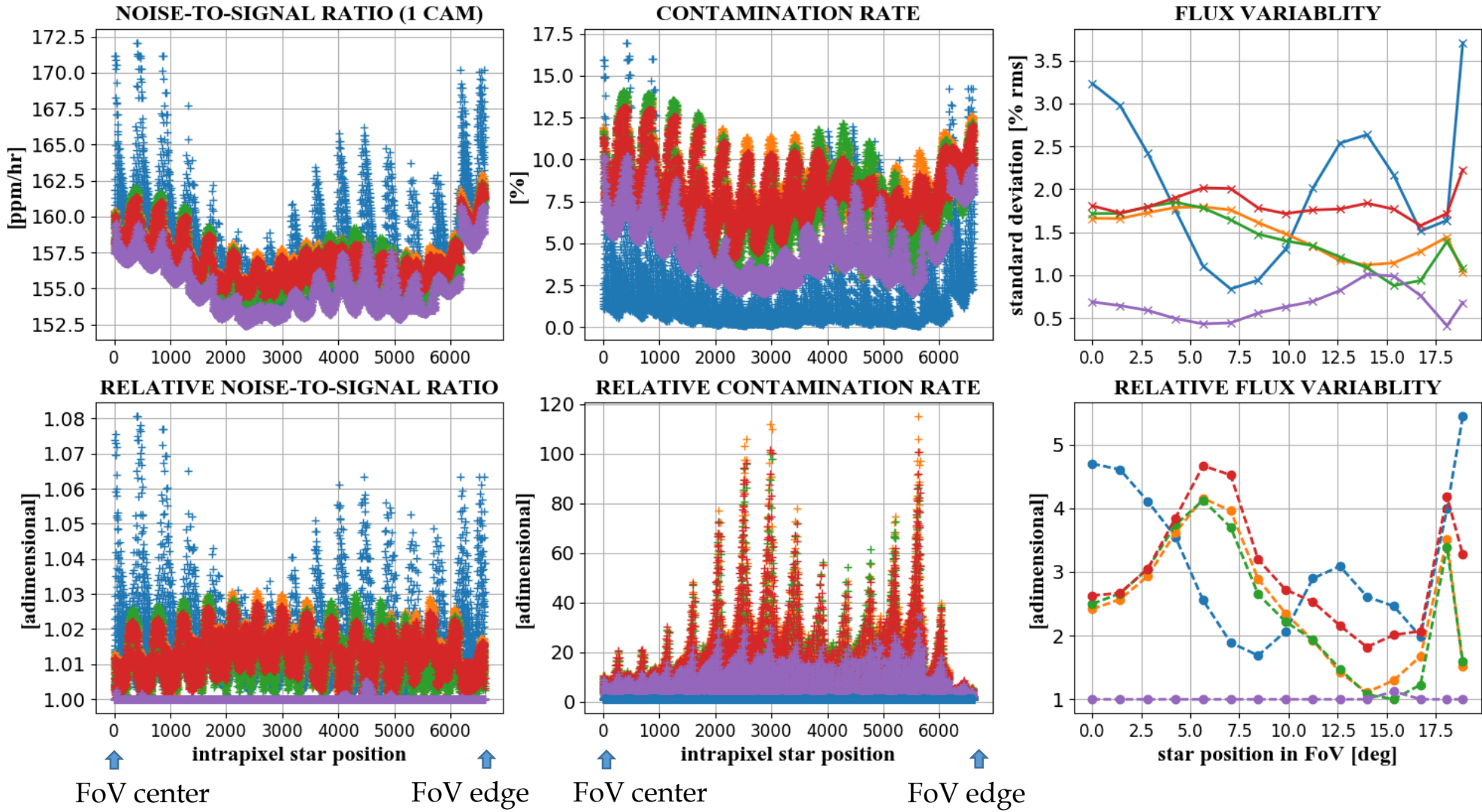
Target star : $m_v = 10.7; T = 6000[K]$; Contaminant star : $m_v = 11.7; T = 6000[K]$
 Euclidian distance between target and contaminant stars : 3.0 pixels



Analysis of masks performances: results

- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask

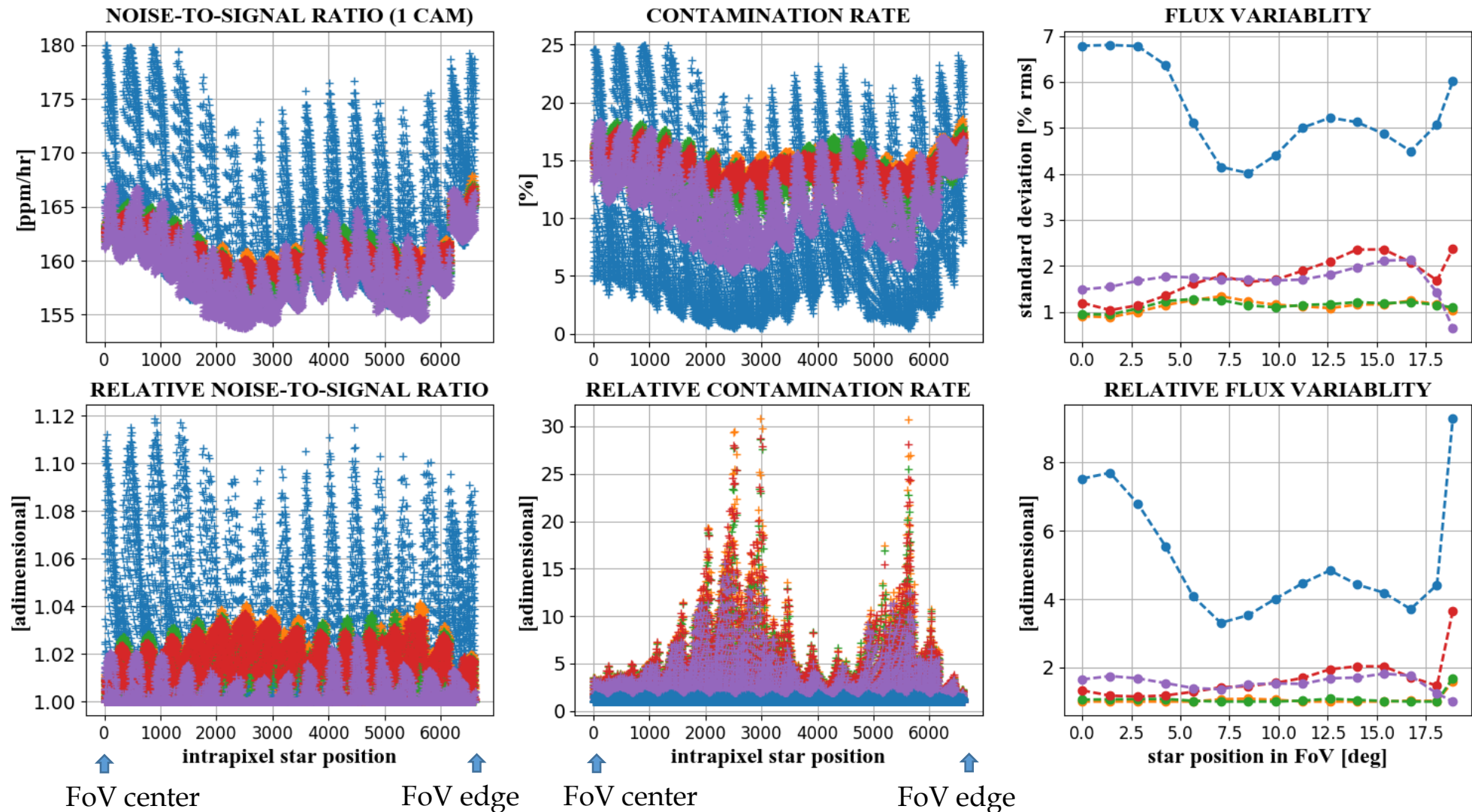
Target star : $m_v = 10.7$; $T = 6000[K]$; Contaminant star : $m_v = 11.7$; $T = 6000[K]$
 Euclidian distance between target and contaminant stars : 2.0 pixels



Analysis of masks performances: results

- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask

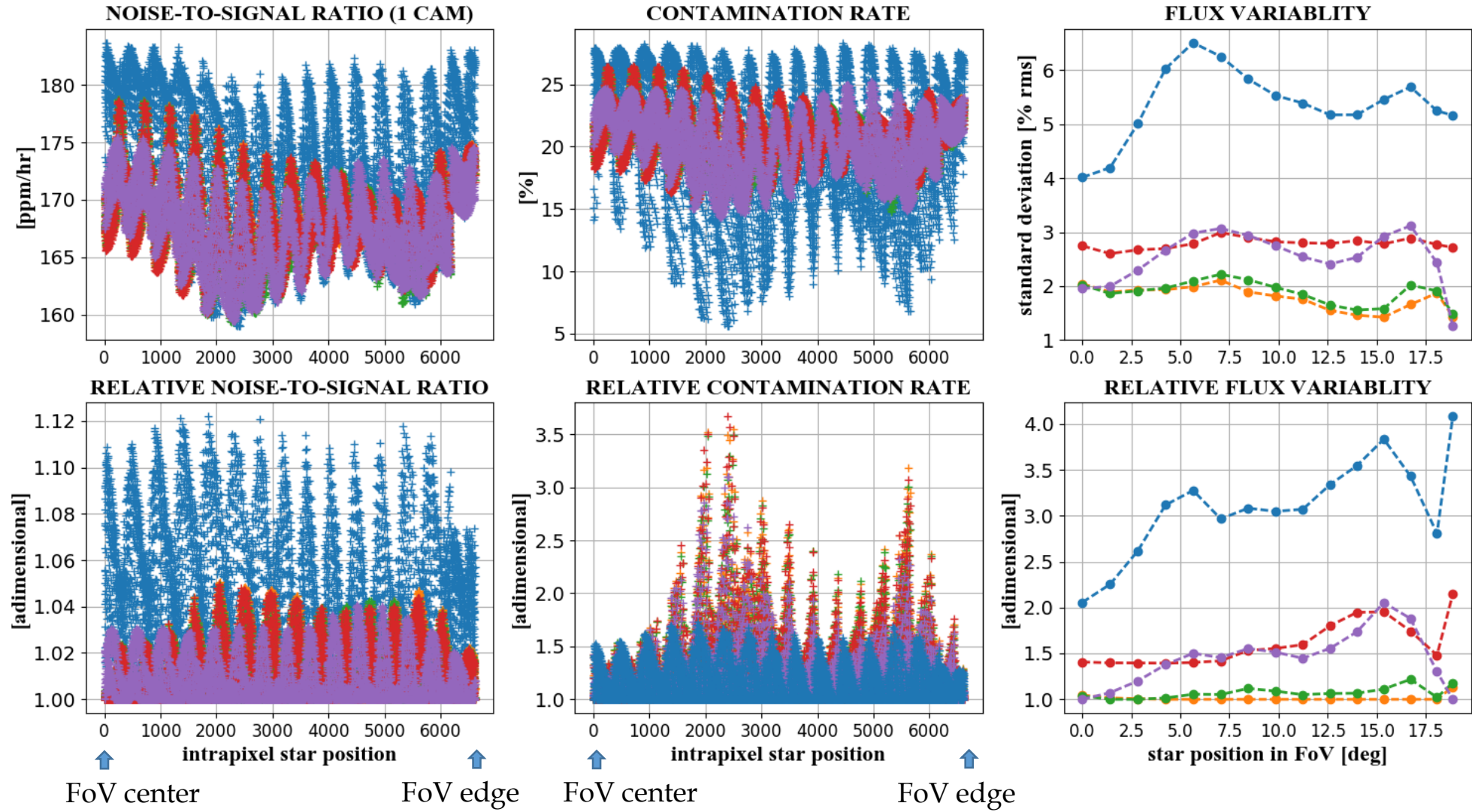
Target star : $m_v = 10.7$; $T = 6000[K]$; Contaminant star : $m_v = 11.7$; $T = 6000[K]$
 Euclidian distance between target and contaminant stars : 1.5 pixel



Analysis of masks performances: results

- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask

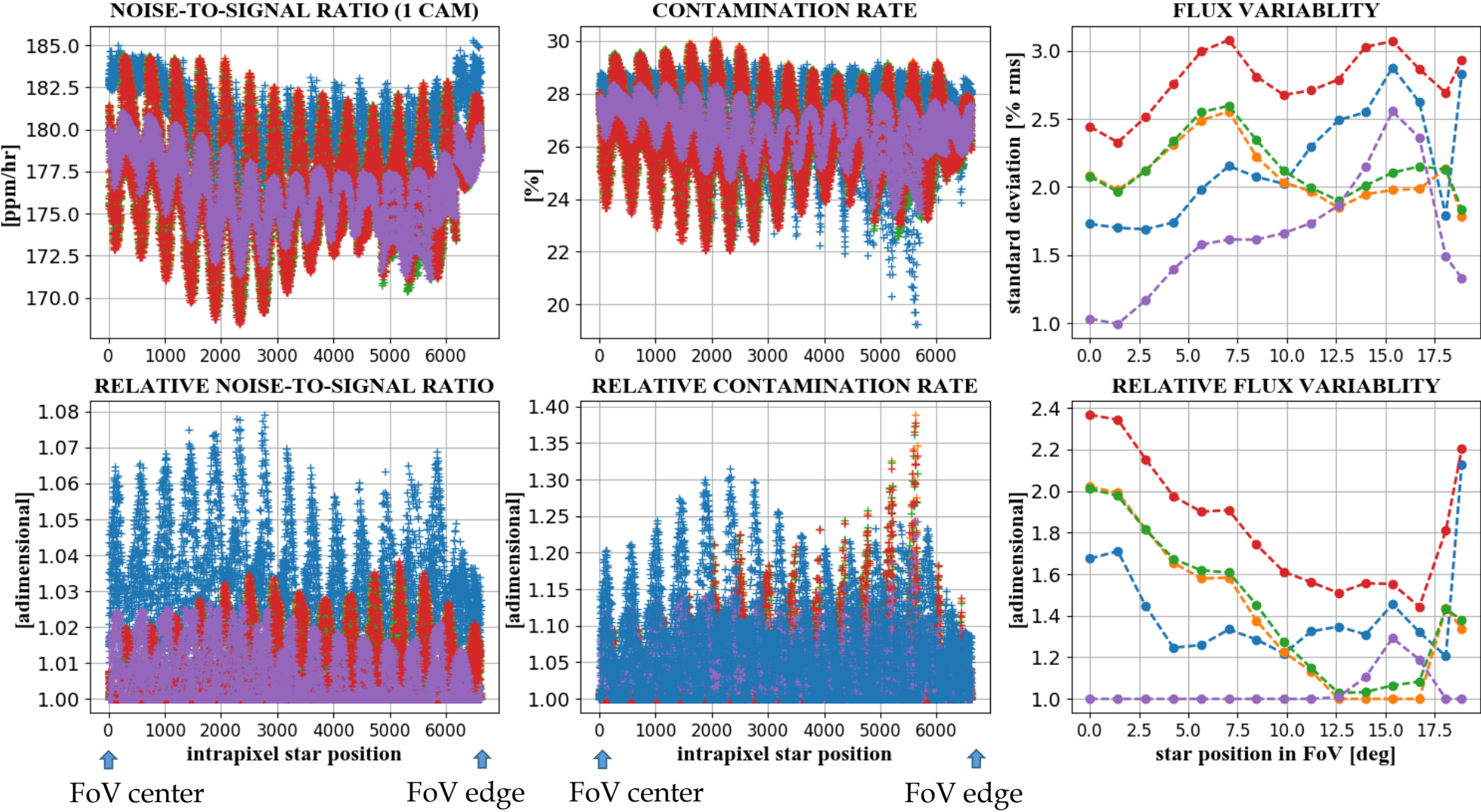
Target star : $m_v = 10.7$; $T = 6000[K]$; Contaminant star : $m_v = 11.7$; $T = 6000[K]$
 Euclidian distance between target and contaminant stars : 1.0 pixel



Analysis of masks performances: results

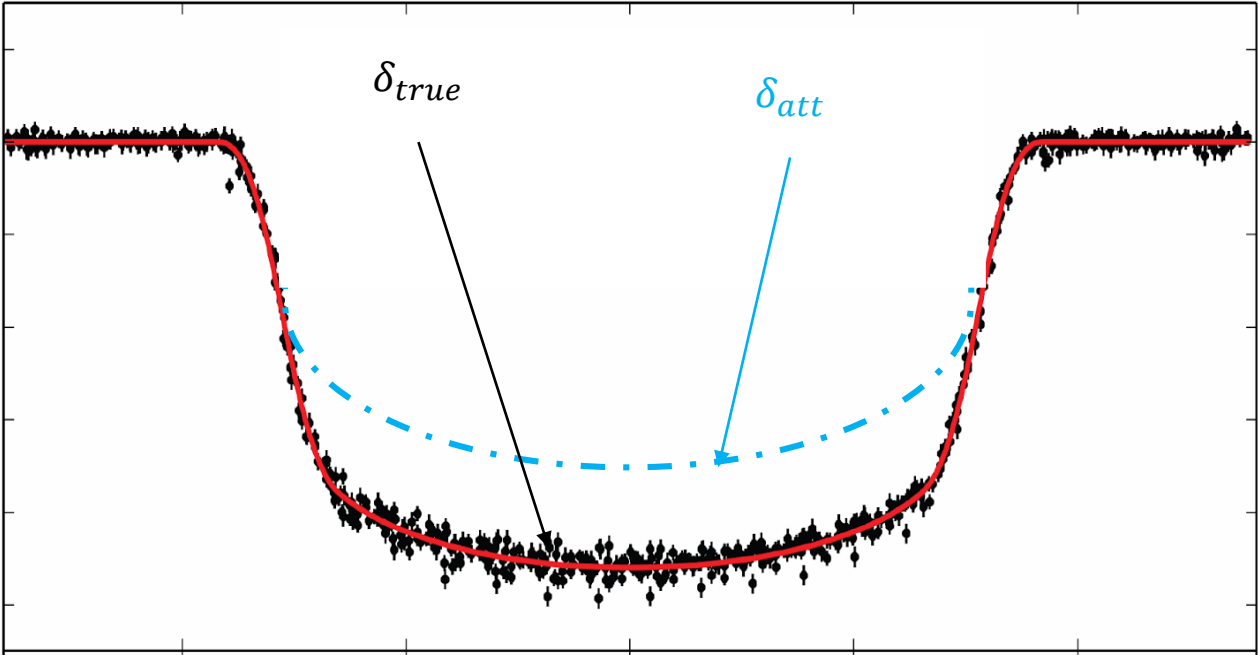
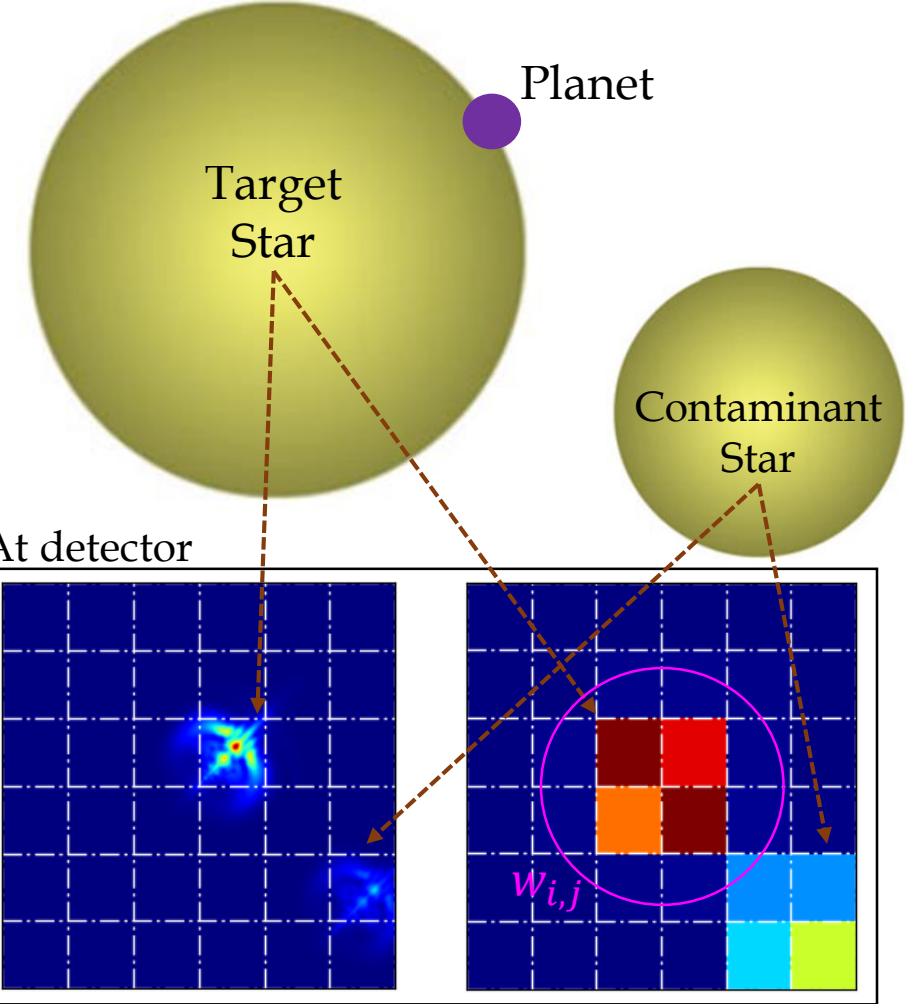
- Minimum NSR Binary Mask
- Minimum NSR Symmetric Gaussian Mask
- Minimum NSR Asymmetric Gaussian Mask
- Fixed Width Gaussian Mask
- Minimum NSR Weights Mask

Target star : $m_v = 10.7; T = 6000[K]$; Contaminant star : $m_v = 11.7; T = 6000[K]$
 Euclidian distance between target and contaminant stars : 0.5 pixel



Conclusions from the presented analysis

Transit attenuation (or transit dilution)



ATTENUATED TRANSIT DEPTH

$$\delta_{att} = (1 - \tau_C) \cdot \delta_{true}$$