Numerical simulations with HADES : improvements & applications to Cepheids

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The outline of my talk

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Radiation hydrodynamics

- Supersonic, hypersonic flows : compression of matter, high temperature, photons emission
- Radiation changes the dynamics and morphology of flows (feedback between hydrodynamics and radiation)
- Radiative flows in stellar physics

Stellar jets ©NASA, ESA, & M. Livio

SNR ©Digitized sky Survey, ESA/ESO/NASA

HADES 2D

- Numerical code for the simulation of radiative hydrodynamics models
- Finite-volume approach
- Parallelized code written in Fortran 90
- Computation of physical quantities as a function of time
- Hydrodynamic quantities
  - Density : ρ
  - Velocity : u
  - Energy : E
- Radiative quantities
  - Radiative energy : E_R
  - Radiative flux : F_R
  - Radiative pressure : P_R

Equations of hydrodynamics

Euler equations :

$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho \mathsf{u}) = 0, \\ \partial_t (\rho \mathsf{u}) + \nabla \cdot (\rho (\mathsf{u} \otimes \mathsf{u}) + p\mathbb{I}) = 0, \\ \partial_t E + \nabla \cdot (\mathsf{u} (E + p)) = 0. \end{cases}$$

Closure of the system : equation of state for ideal gas :

$$p = (\gamma - 1) \left(E - \frac{1}{2} \rho \mathsf{u}^2 \right),$$

with γ the adiabatic index.

Equations of radiative transfer

Equations of radiative transfer with photon-matter interaction :

$$\begin{cases} \partial_t E_R + \nabla \cdot \mathsf{F}_R = -c\, S^0, \\ \partial_t \left(c^{-2}\mathsf{F}_R\right) + \nabla \cdot \mathsf{P}_R = -\mathsf{S}. \end{cases}$$

Source terms (LTE) :

$$S^0 = \kappa_P \left(E_R - a_R T^4\right),$$
$$\mathsf{S} = \kappa_R\, \mathsf{F}_R / c.$$

- Planck mean opacity :

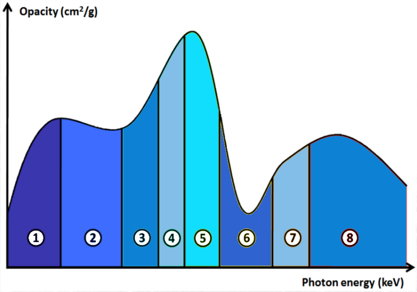
$$\kappa_P = \frac{\int_\nu \kappa(\nu) B(\nu, T)\, \mathrm{d}\nu}{\int_\nu B(\nu, T)\, \mathrm{d}\nu}$$

- Rosseland mean opacity :

$$\kappa_R^{-1} = \frac{\int_\nu \chi^{-1}(\nu) \partial_T B(\nu, T)\, \mathrm{d}\nu}{\int_\nu \partial_T B(\nu, T)\, \mathrm{d}\nu}.$$

Equations of radiative transfer

- Generally, strong fluctuation of opacities



- Multigroup strategy : segmentation of frequencies in $\mathcal{G}$ groups and calculation of radiative quantities for each group $g = 1, \ldots, \mathcal{G}$

Equations of radiation hydrodynamics

- Strategy : Euler equations with multigroup model coupling

Radiative hydrodynamics equations :

$$
\text{Coupling}\begin{cases}
\text{Hydro}\begin{cases}
\partial_t \rho + \nabla\cdot(\rho \mathsf{u}) = 0,\\
\partial_t(\rho\mathsf{u}) + \nabla\cdot(\rho(\mathsf{u}\otimes\mathsf{u}) + p\mathbb{I}) = \sum_{g=1}^{\mathcal{G}} \mathsf{S}_g,\\
\partial_t E + \nabla\cdot(\mathsf{u}(E+p)) = \sum_{g=1}^{\mathcal{G}} c\, S_g^0,
\end{cases}\\
\text{Rad}\begin{cases}
\partial_t E_{R_g} + \nabla\cdot \mathsf{F}_{R_g} = -c\, S_g^0, & g = 1,\dots,\mathcal{G},\\
\partial_t\left(c^{-2}\mathsf{F}_{R_g}\right) + \nabla\cdot \mathsf{P}_{R_g} = -\mathsf{S}_g, & g = 1,\dots,\mathcal{G}.
\end{cases}
\end{cases}
$$

Numerical methods in HADES

System of equations of the form :

$$\frac{\partial \mathrm{q}}{\partial t} + \mathrm{div}\ \mathrm{f}(\mathrm{q}) = \mathrm{s}(t, \mathrm{q}).$$

Intervention of two subsystems :

- Homogeneous partial differential system of equations

$$\frac{\partial \mathrm{q}}{\partial t} + \mathrm{div}\ \mathrm{f}(\mathrm{q}) = 0.$$

- Ordinary differential system of equations

$$\frac{\mathrm{dq}}{\mathrm{d}t} = \mathrm{s}(t, \mathrm{q}).$$

Numerical methods in HADES

- Homogeneous PDE :
  - 2D finite-volume
  - Directional splitting
  - MUSCL-Hancock scheme
  - HLL, HLLC, HLLE solvers for flux computation
- ODE
  - Explicit and implicit schemes : Euler methods, midpoint method, Runge-Kutta methods
- Opacities handling :
  - Tables of opacities : results from atomic physics calculations
  - Opacity calculations : interpolation on the grid (ρ, T)

The outline of my talk

Radiative transfer : two asymptotic cases

λ : mean free path of a photon
L : characteristic length of the issuing medium

- Optical thin medium : λ is very long with respect to the characteristic length L of the issuing medium ($\lambda/L \gg 1$)
  - Weak interaction between photons and the medium
  - Energy loss measured by a cooling function
  - Case already included in the HADES code
- Optical thick medium : $\lambda/L < 1$
- Optical very thick medium : λ is very short with respect to the characteristic length L of the issuing medium ($\lambda/L \ll 1$).
  - Fluid opaque to photons
  - Local radiative phenomena
  - Diffusion approximation

Description of radiative transfer much simpler. Faster calculations to comprehend physics

Diffusion approximation

- Radiative quantities in the diffusion approximation :
  - $E_R = a_R T^4$
  - $\mathrm{F}_R = -\frac{1}{3}\frac{c}{\kappa_R}\nabla E_R$
  - $\mathrm{P}_R = \frac{1}{3}E_R\mathbb{I}$

Equations of the diffusion approximation :

$$\partial_t \rho + \nabla\cdot(\rho\mathrm{u}) = 0,$$
$$\partial_t(\rho\mathrm{u}) + \nabla\cdot(\rho(\mathrm{u}\otimes\mathrm{u}) + p\mathbb{I}) = -\frac{1}{3}\nabla E_R,$$
$$\partial_t E + \nabla\cdot(\mathrm{u}(E+p)) = -\partial_t E_R + \frac{1}{3}\frac{c}{\kappa_R}\Delta E_R - \frac{4}{3}\nabla\cdot(\mathrm{u}E_R).$$

- Diffusion term in the third equation

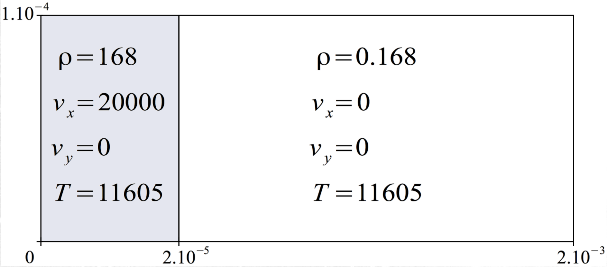
Diffusion approximation

- Finite difference method for the computation of source terms
- Considering the finite volume equation written at spatial cell (i,j) and at time t^n, the source term is given by :

$$S = \begin{bmatrix} 0 \\ \dfrac{-1}{3}\dfrac{E^n_{R_{i+1,j}} - E^n_{R_{i-1,j}}}{2\Delta x} \\ \dfrac{-1}{3}\dfrac{E^n_{R_{i,j+1}} - E^n_{R_{i,j-1}}}{2\Delta y} \\ -\dfrac{E^n_{R_{i,j}} - E^{n-1}_{R_{i,j}}}{\Delta t} + \dfrac{1}{3}\dfrac{c}{\kappa_R}\left[\dfrac{E^n_{R_{i+1,j}} - 2E^n_{R_{i,j}} + E^n_{R_{i-1,j}}}{(\Delta x)^2} + \dfrac{E^n_{R_{i,j+1}} - 2E^n_{R_{i,j}} + E^n_{R_{i,j-1}}}{(\Delta y)^2}\right] - \dots \\ \dots \dfrac{4}{3}\left[\dfrac{\mathsf{u}^n_{i+1,j}E^n_{R_{i+1,j}} - \mathsf{u}^n_{i-1,j}E^n_{R_{i-1,j}}}{2\Delta x} + \dfrac{\mathsf{u}^n_{i,j+1}E^n_{R_{i,j+1}} - \mathsf{u}^n_{i,j-1}E^n_{r_{i,j-1}}}{2\Delta y}\right]. \end{bmatrix}$$

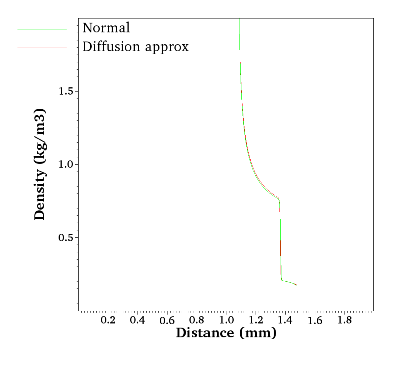
Code validation

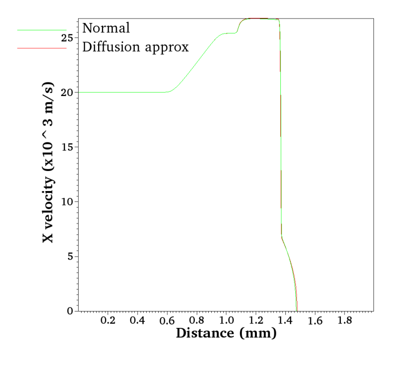
- Benchmark test : stationary radiative shock



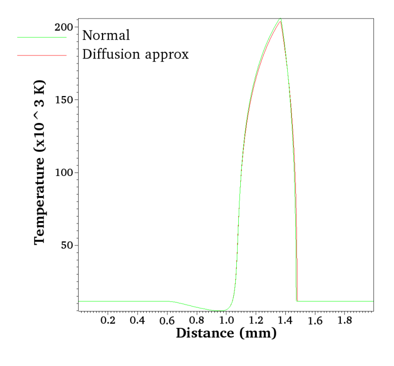
- $M_x = 2000$, $M_y = 10$ and $T_f = 4.10^{-8}$
- Mean free path : λ = 1 micron

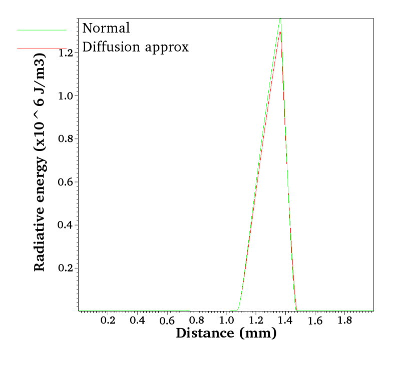
Results


Normal
Diffusion approx
Density (kg/m3)
1.5
1.0
0.5
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Distance (mm)


Normal
Diffusion approx
X velocity (x10 ^ 3 m/s)
25
20
15
10
5
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
Distance (mm)

Results


Normal
Diffusion approx
Temperature (x10 ^ 3 K)
200
150
100
50
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8
Distance (mm)


Normal
Diffusion approx
Radiative energy (x10 ^ 6 J/m3)
1.2
1.0
0.8
0.6
0.4
0.2
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8
Distance (mm)

The outline of my talk

Pulsating stars : Cepheids

- 1784 : 1st Cepheid δ Cephei discovered by J. Goodricke
- Yellow or red supergiant
- Stars with a regularly varying luminosity (P-L relationship discovered by H. Leavitt in 1912)

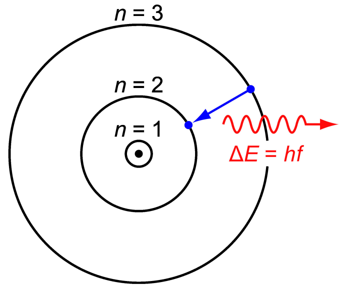
$$M = a(\log(P) - 1) + b$$

- Distance indicators for extragalactic astronomy

FIGURE : RS Puppis

Cepheids and $H\alpha$ emissions

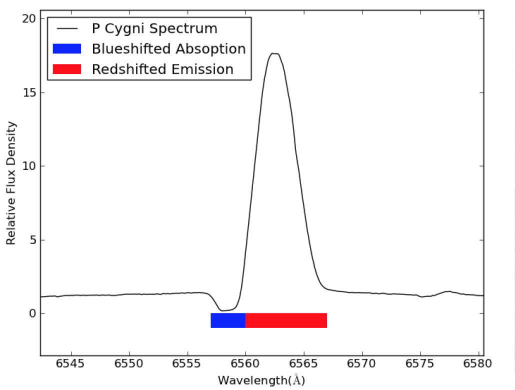
- $H\alpha$: spectral line in the deep-red visible created by hydrogen with a wavelength of 656.28 nm
- When hydrogen electron falls from third ($n = 3$) to second ($n = 2$) lowest energy level

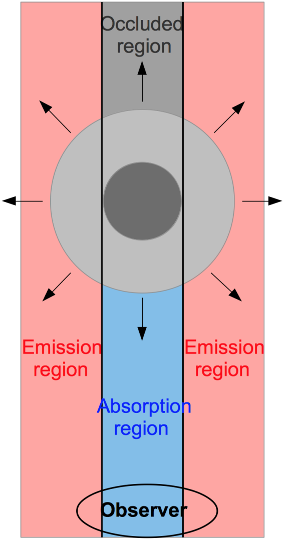


- Strong $H\alpha$ asymmetry for long-period Cepheids. Asymmetric P Cygni profile

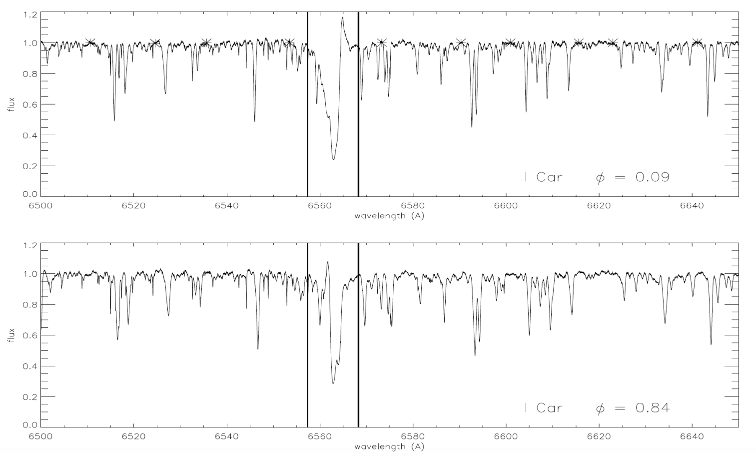
P Cygni profile

Common characteristic of stars with a high mass ejection or violent stellar winds (long-period Cepheids up to $10^{-5}\ M_{\odot}$/year)





Example : l Carinæ (l Car)



Central absorption composant and emission composants (redshift and blueshift). Strong asymmetry

Work purpose

- Normally : spectral lines of a star absorption produced by its calm atmosphere
- P Cygni profile observed for stars which present stellar winds
- $H\alpha$ asymmetries in long-period Cepheid profiles may be caused by the presence of strong shocks

Purpose : use of numerical tool to perform simulations of shocks in Cepheid envelopes and to reconstruct observables in order to compare with observations

Ideal candidate : l Carinæ (l Car)

- Readily visible to the naked eye Cepheid in the Carina constellation
- The higher apparent angular diameter : more accurate spectro - interferometric measures

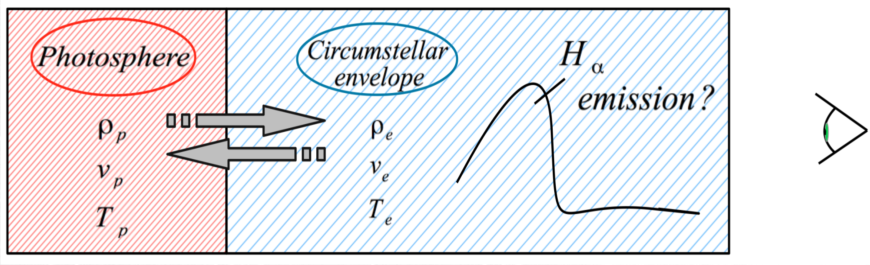
Period	Mass	Radius	Temperature	Radial velocity
35.560 j	$8.4\text{–}13\ M_\odot$	$\approx 180\ R_\odot$	5091 K	39 km/s

- CE around 10–100 AU with an average temperature of 100 K
- Velocity shock estimation : $v_{shock} = 100$ km/s *(N.Nardetto et al. 08)*
- Mach number

$$M = \frac{v_{shock}}{c} \approx 85$$

with c the sound velocity in the CE : $c = \sqrt{(\gamma p)/\rho}$

Modelling



Following of the local opacity evolution depending on density and temperature of the envelope

Work steps

1. Reconstruction of an observable from the physical quantities
2. Hydrodynamic simulations :
   - Envelope including constant density as well as density gradients
   - Envelope first at rest and then driven by stellar wind
   - Velocity field of the photosphere : ideal sinusoidal pulsation and then more realistic profiles
3. Radiation hydrodynamics simulations :
   - First step : diffusion approximation to prepare tables of opacities
   - Second step : full radiative transfer with the multigroup model (including one at the $H\alpha$ line)
4. Quantitative comparison of the numerical results with P. Kervella et al. observations

Thank you for your attention !